

Electroweak Theory and Higgs Physics

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<http://boudin.fnal.gov/AcLec/AcLecQuigg.html>

A Decade of Discovery Past . . .

- ▷ Electroweak theory → law of nature
- ▷ Higgs-boson influence observed in the vacuum
- ▷ Neutrino flavor oscillations: $\nu_\mu \rightarrow \nu_\tau$,
 $\nu_e \rightarrow \nu_\mu / \nu_\tau$
- ▷ Understanding QCD
- ▷ Discovery of top quark
- ▷ Direct \mathcal{CP} violation in $K \rightarrow \pi\pi$
- ▷ B -meson decays violate \mathcal{CP}
- ▷ Flat universe dominated by dark matter, energy
- ▷ Detection of ν_τ interactions
- ▷ Quarks & leptons structureless at TeV scale

A Decade of Discovery Past . . .

- ▷ Electroweak theory → law of nature
[Z , e^+e^- , $\bar{p}p$, νN , $(g - 2)_\mu$, . . .]
- ▷ Higgs-boson influence observed in the vacuum
[EW experiments]
- ▷ Neutrino flavor oscillations: $\nu_\mu \rightarrow \nu_\tau$,
 $\nu_e \rightarrow \nu_\mu/\nu_\tau$ [ν_\odot , ν_{atm} , reactors]
- ▷ Understanding QCD
[heavy flavor, Z^0 , $\bar{p}p$, νN , ep , ions, lattice]
- ▷ Discovery of top quark [$\bar{p}p$]
- ▷ Direct \mathcal{CP} violation in $K \rightarrow \pi\pi$ [fixed-target]
- ▷ B -meson decays violate \mathcal{CP} [$e^+e^- \rightarrow B\bar{B}$]
- ▷ Flat universe dominated by dark matter, energy
[SN Ia, CMB, LSS]
- ▷ Detection of ν_τ interactions [fixed-target]
- ▷ Quarks & leptons structureless at TeV scale
[mainly colliders]

Goal: Understanding the Everyday

- ▷ Why are there atoms?
- ▷ Why chemistry?
- ▷ Why stable structures?
- ▷ What makes life possible?

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*What would the world be like,
without a (Higgs) mechanism to hide
electroweak symmetry and give
masses to the quarks and leptons?*

Searching for the mechanism of electroweak symmetry breaking, we seek to understand

why the world is the way it is.

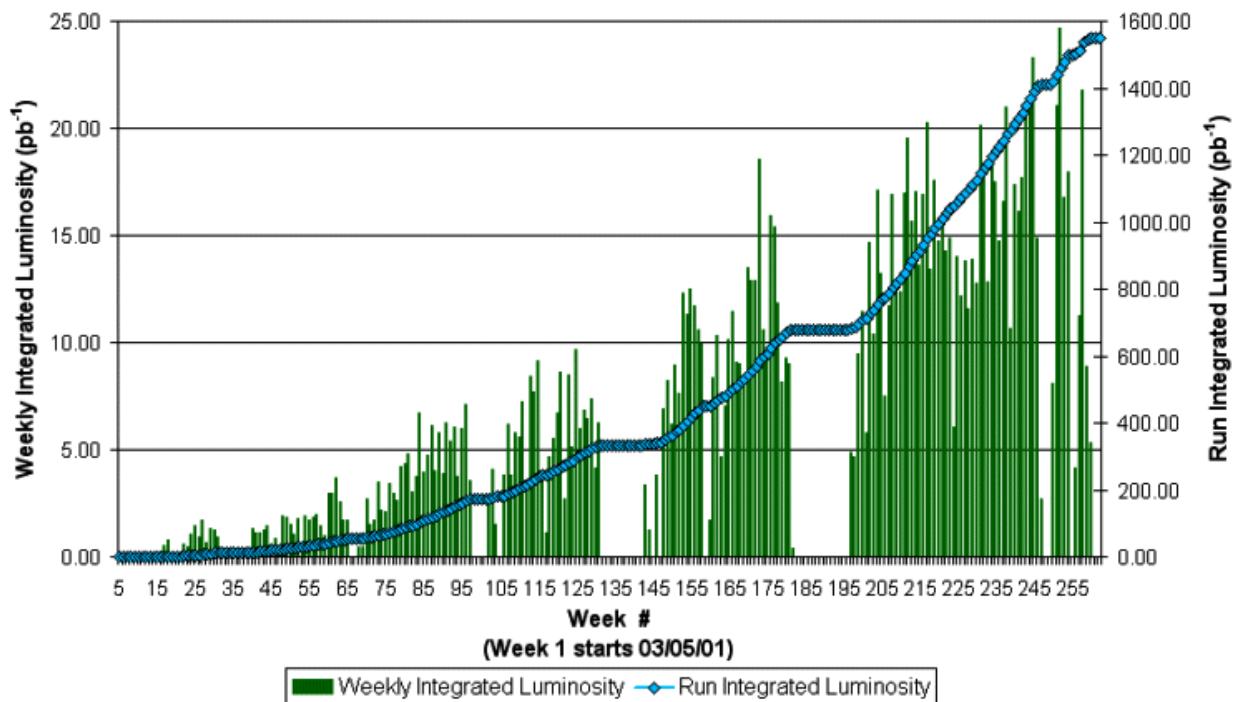
This is one of the deepest questions humans have ever pursued, and

it is coming within the reach of particle physics.

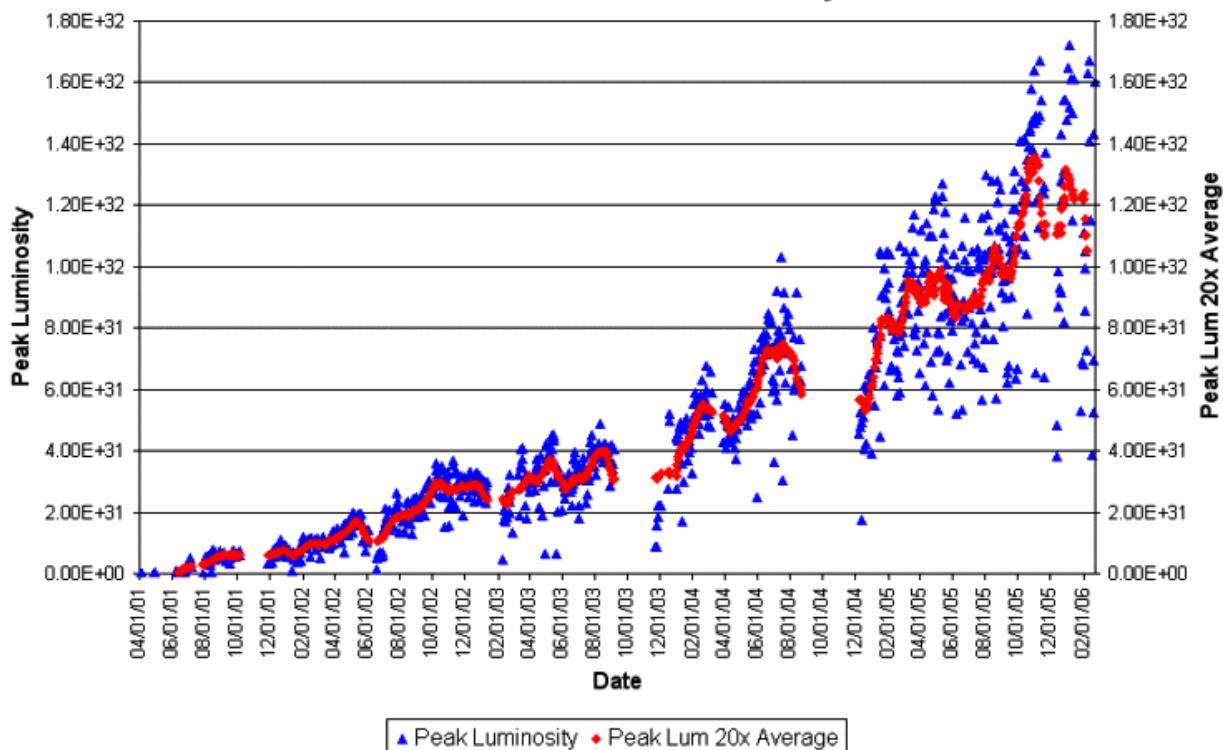
Tevatron Collider is running *now*,
breaking new ground in sensitivity



Collider Run II Integrated Luminosity



Collider Run II Peak Luminosity



Tevatron Collider in a Nutshell

980-GeV protons, antiprotons
 $(2\pi \text{ km})$

frequency of revolution $\approx 45\,000 \text{ s}^{-1}$

392 ns between crossings
 $(36 \otimes 36 \text{ bunches})$

collision rate $= \mathcal{L} \cdot \sigma_{\text{inelastic}} \approx 10^7 \text{ s}^{-1}$

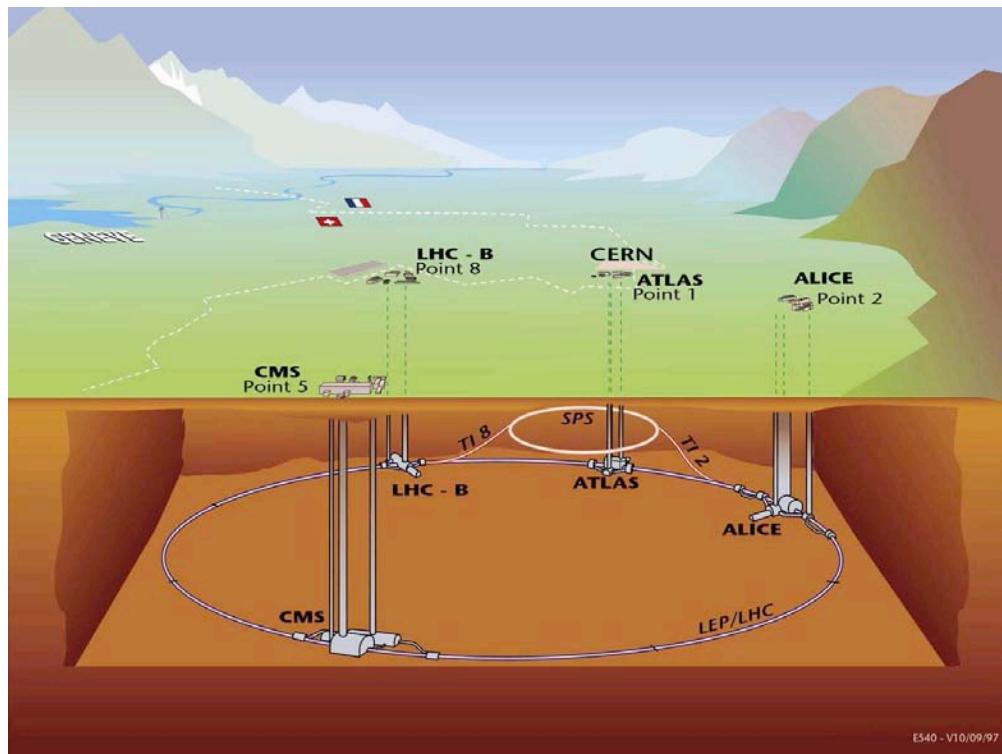
$c \approx 10^9 \text{ km/h}; \quad v_p \approx c - 495 \text{ km/h}$

Record $\mathcal{L}_{\text{init}} = 1.64 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$

[CERN ISR: $pp, 1.4$]

Maximum \bar{p} at Low β : 1.661×10^{12}

The LHC will operate *soon*, breaking new ground in energy and sensitivity



LHC in a nutshell

7-TeV protons on protons (27 km);

$$v_p \approx c - 10 \text{ km/h}$$

Novel two-in-one dipoles (≈ 9 teslas)

Startup: $43 \otimes 43 \rightarrow 156 \otimes 156$
bunches, $\mathcal{L} \approx 6 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$

Early: 936 bunches,
 $\mathcal{L} \gtrsim 5 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ [75 ns]

Next phase: 2808 bunches,

$$\mathcal{L} \rightarrow 2 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$$

25 ns bunch spacing

Eventual: $\mathcal{L} \gtrsim 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$:
 $100 \text{ fb}^{-1}/\text{year}$

Tentative Outline . . .

▷ $SU(2)_L \otimes U(1)_Y$ theory

Gauge theories

Spontaneous symmetry breaking

Consequences: W^\pm , Z^0/NC , H , m_f ?

Measuring $\sin^2 \theta_W$ in νe scattering

GIM / CKM

▷ Phenomena at tree level and beyond

Z^0 pole

W mass and width

Vacuum energy problem

... Outline

- ▷ The Higgs boson and the 1-TeV scale
 - Why the Higgs boson must exist
 - Higgs properties, constraints
 - How well can we anticipate M_H ?
 - Higgs searches
- ▷ The problems of mass
- ▷ The EW scale and beyond
 - Hierarchy problem
 - Why is the EW scale so small?
 - Why is the Planck scale so large?
- ▷ Outlook

General References

- ▷ C. Quigg, “Nature’s Greatest Puzzles,” hep-ph/0502070
- ▷ C. Quigg, “The Electroweak Theory,” hep-ph/0204104 (TASI 2000 Lectures)
- ▷ C. Quigg, *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions*
- ▷ I. J. R. Aitchison & A. J. G. Hey, *Gauge Theories in Particle Physics*
- ▷ R. N. Cahn & G. Goldhaber, *Experimental Foundations of Particle Physics*
- ▷ G. Altarelli & M. Grünewald, “Precision Electroweak Tests of the SM,” hep-ph/0404165
- ▷ F. Teubert, “Electroweak Physics,” ICHEP04
- ▷ S. de Jong, “Tests of the Electroweak Sector of the Standard Model,” EPS HEPP 2005

Problem sets: <http://lutece.fnal.gov/TASI/default.html>

Our picture of matter

Pointlike constituents ($r < 10^{-18}$ m)

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$$

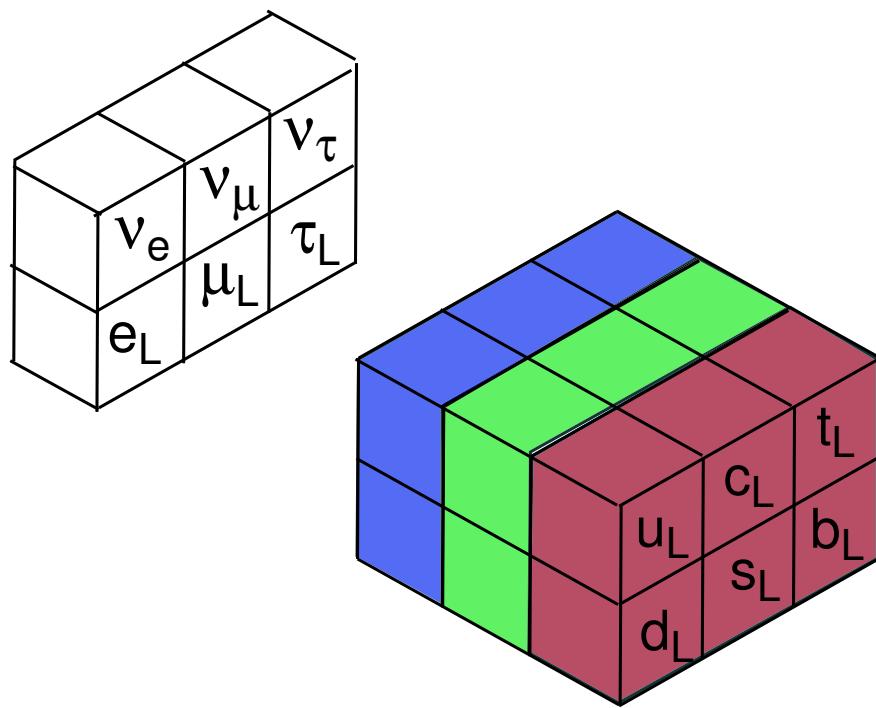
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

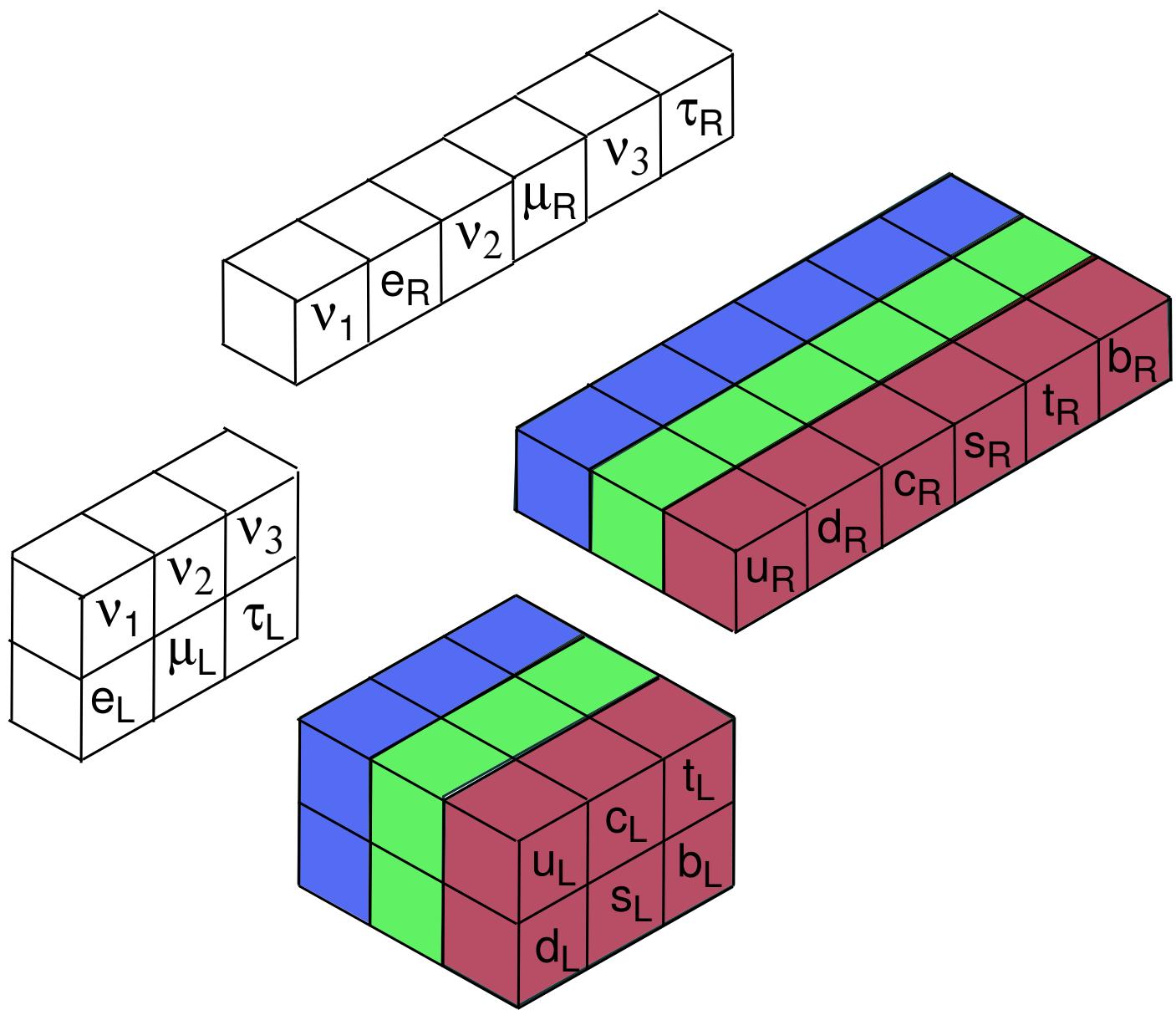
Few fundamental forces, derived from gauge symmetries

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

Electroweak symmetry breaking

Higgs mechanism?





SYMMETRIES \Rightarrow INTERACTIONS

Phase Invariance (Symmetry) in Quantum Mechanics

QM STATE: COMPLEX SCHRÖDINGER WAVE
FUNCTION $\psi(x)$

OBSERVABLES

$$\langle O \rangle = \int d^n x \psi^* O \psi$$

ARE UNCHANGED

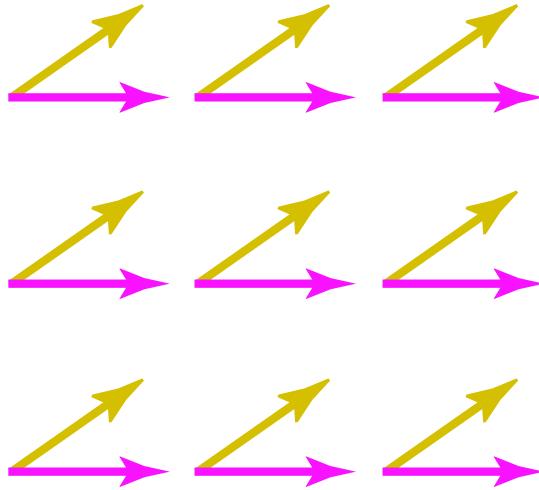
UNDER A GLOBAL PHASE ROTATION

$$\begin{aligned}\psi(x) &\rightarrow e^{i\theta} \psi(x) \\ \psi^*(x) &\rightarrow e^{-i\theta} \psi^*(x)\end{aligned}$$

- Absolute phase of the wave function cannot be measured (is a matter of convention).
- Relative phases (interference experiments) are unaffected by a global phase rotation.

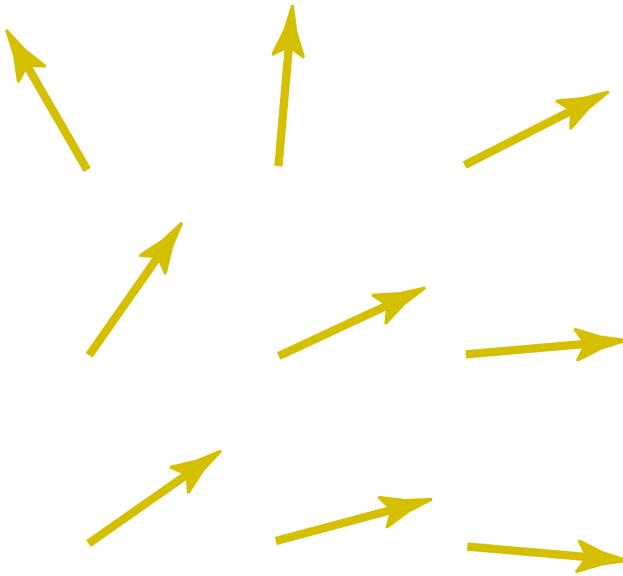


GLOBAL ROTATION — SAME EVERYWHERE



MIGHT WE CHOOSE ONE PHASE CONVENTION
IN RIO AND ANOTHER IN BATAVIA?

A DIFFERENT CONVENTION AT EACH POINT?



$$\psi(x) \rightarrow e^{iq\alpha(x)}\psi(x)$$

THERE IS A PRICE.

Some variables (e.g., momentum) and the Schrödinger equation itself contain derivatives.
Under the transformation

$$\psi(x) \rightarrow e^{iq\alpha(x)}\psi(x)$$

the gradient of the wave function transforms as

$$\nabla\psi(x) \rightarrow e^{iq\alpha(x)}[\nabla\psi(x) + iq(\nabla\alpha(x))\psi(x)]$$

The $\nabla\alpha(x)$ term spoils local phase invariance.

TO RESTORE LOCAL PHASE INVARIANCE ...

Modify the equations of motion and observables.

Replace ∇ by $\nabla + iq\vec{A}$

“Gauge-covariant derivative”

If the vector potential \vec{A} transforms under local phase rotations as

$$\vec{A}(x) \rightarrow \vec{A}'(x) \equiv \vec{A}(x) - \nabla\alpha(x),$$

then $(\nabla + iq\vec{A})\psi \rightarrow e^{iq\alpha(x)}(\nabla + iq\vec{A})\psi$ and
 $\psi^*(\nabla + iq\vec{A})\psi$ is invariant under local rotations.

NOTE . . .

- $\vec{A}(x) \rightarrow \vec{A}'(x) \equiv \vec{A}(x) - \nabla\alpha(x)$ has the form of a gauge transformation in electrodynamics.
- The replacement $\nabla \rightarrow (\nabla + iq\vec{A})$ corresponds to $\vec{p} \rightarrow \vec{p} - q\vec{A}$

FORM OF INTERACTION IS DEDUCED
FROM LOCAL PHASE INVARIANCE

⇒ MAXWELL'S EQUATIONS

DERIVED
FROM A SYMMETRY PRINCIPLE

QED is the gauge theory based on
 $U(1)$ phase symmetry

GENERAL PROCEDURE

- Recognize a symmetry of Nature.
- Build it into the laws of physics.
(Connection with conservation laws)
- Impose symmetry in stricter (local) form.

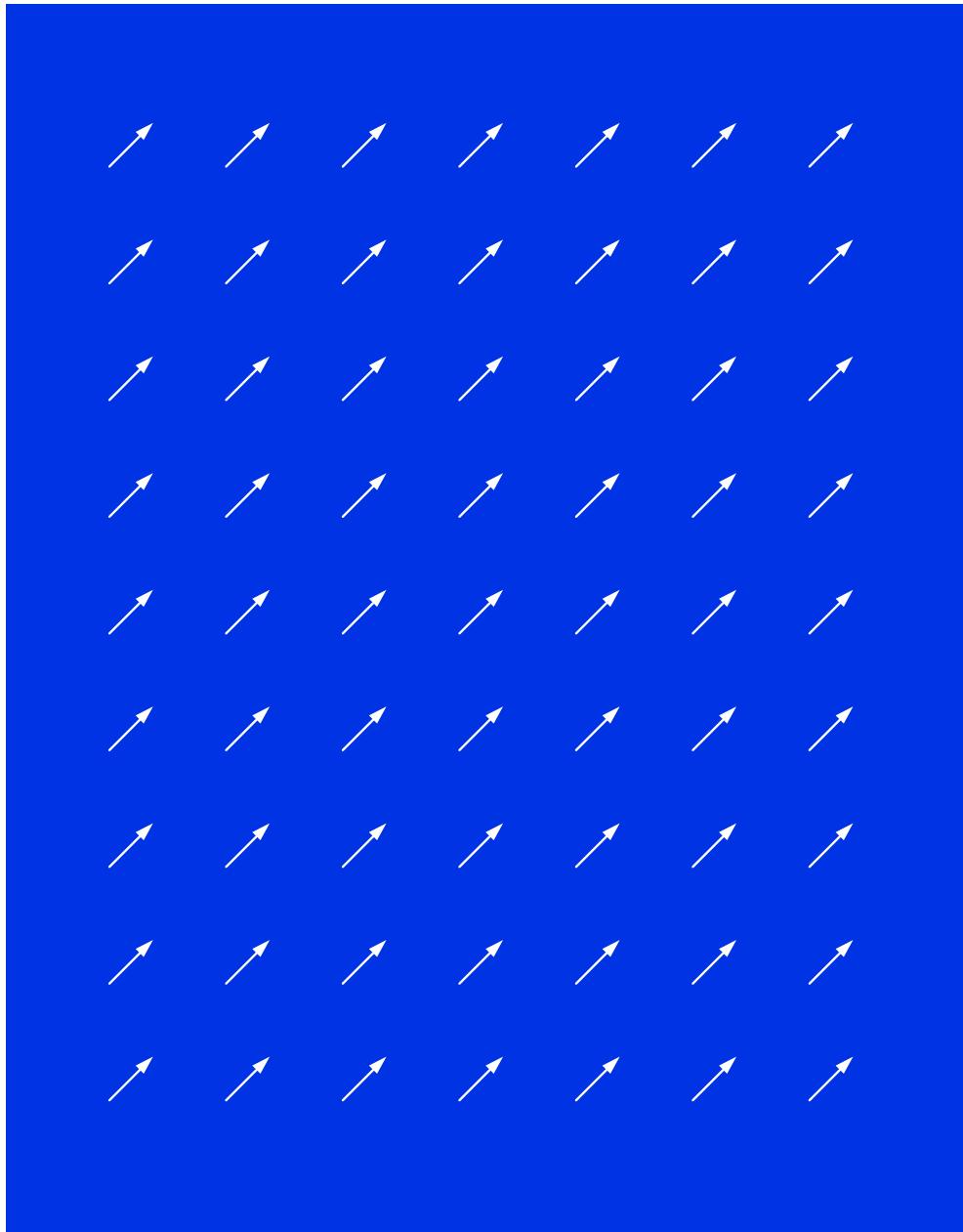
⇒ INTERACTIONS

- Massless vector fields (gauge fields)
- Minimal coupling to the conserved current
- Interactions among the gauge fields, if symmetry is non-Abelian

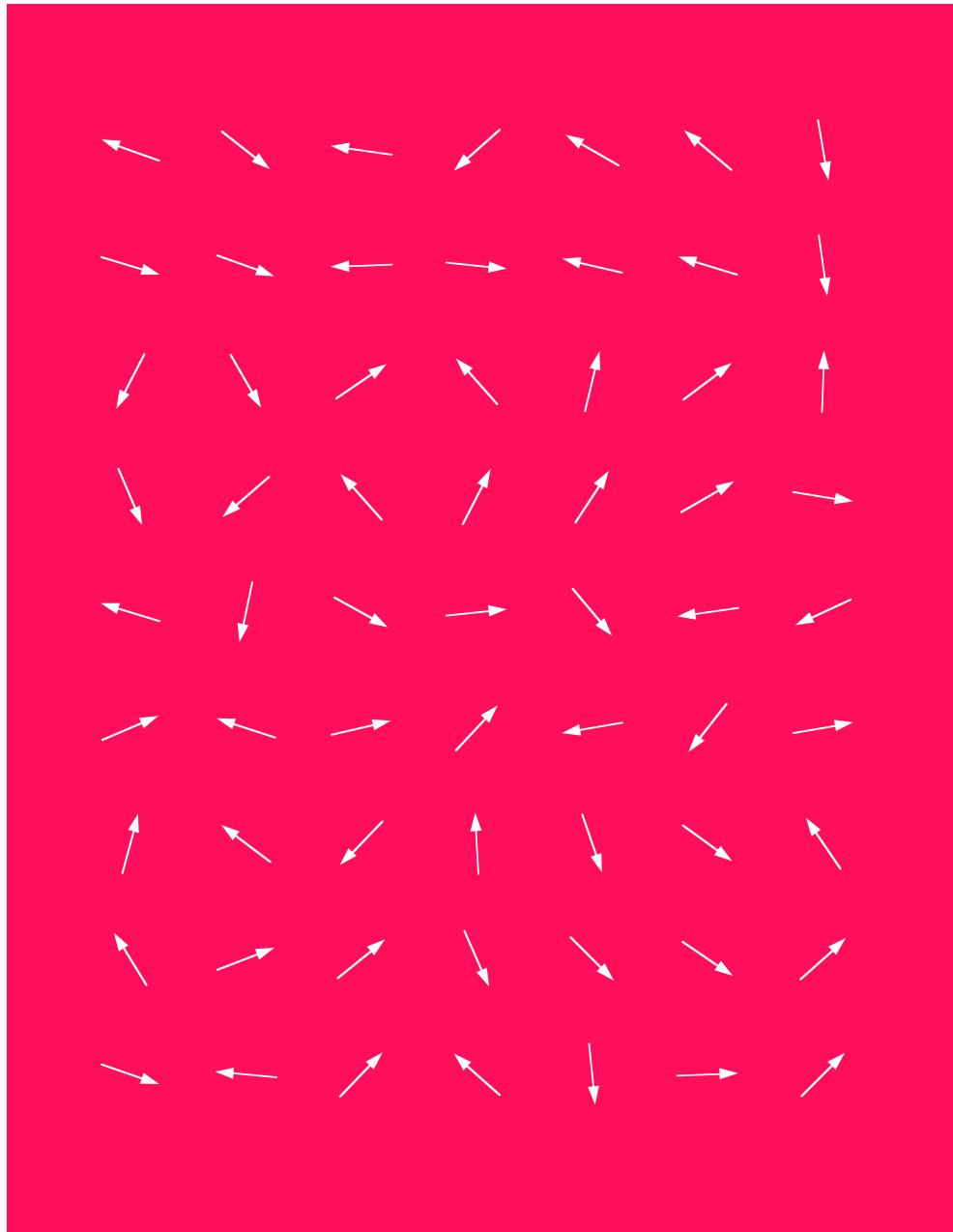
Posed as a problem in mathematics, construction of a gauge theory is always possible (at the level of a classical \mathcal{L} ; consistent quantum theory may require additional vigilance).

Formalism is no guarantee that the gauge symmetry was chosen wisely.

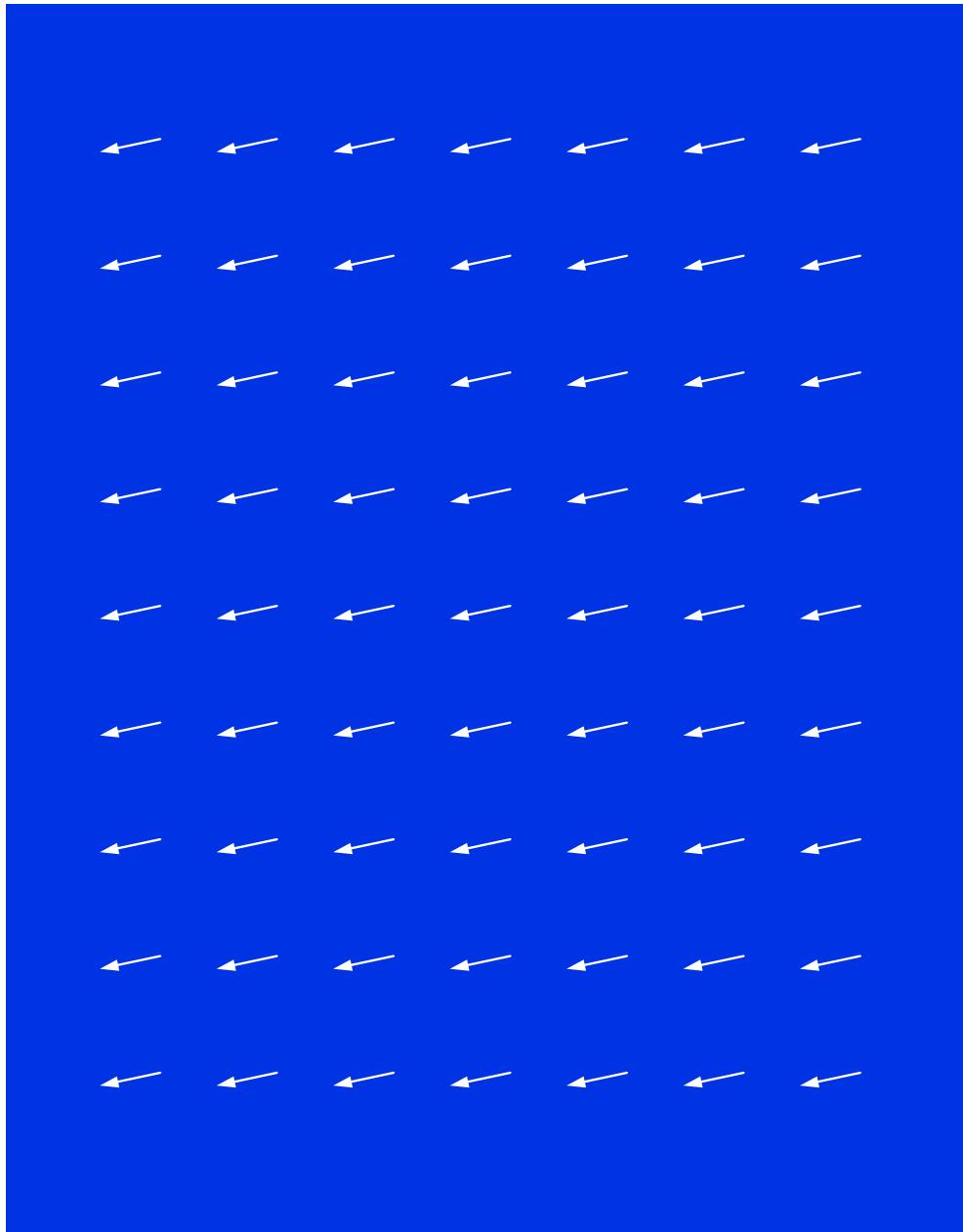
The Crystal World



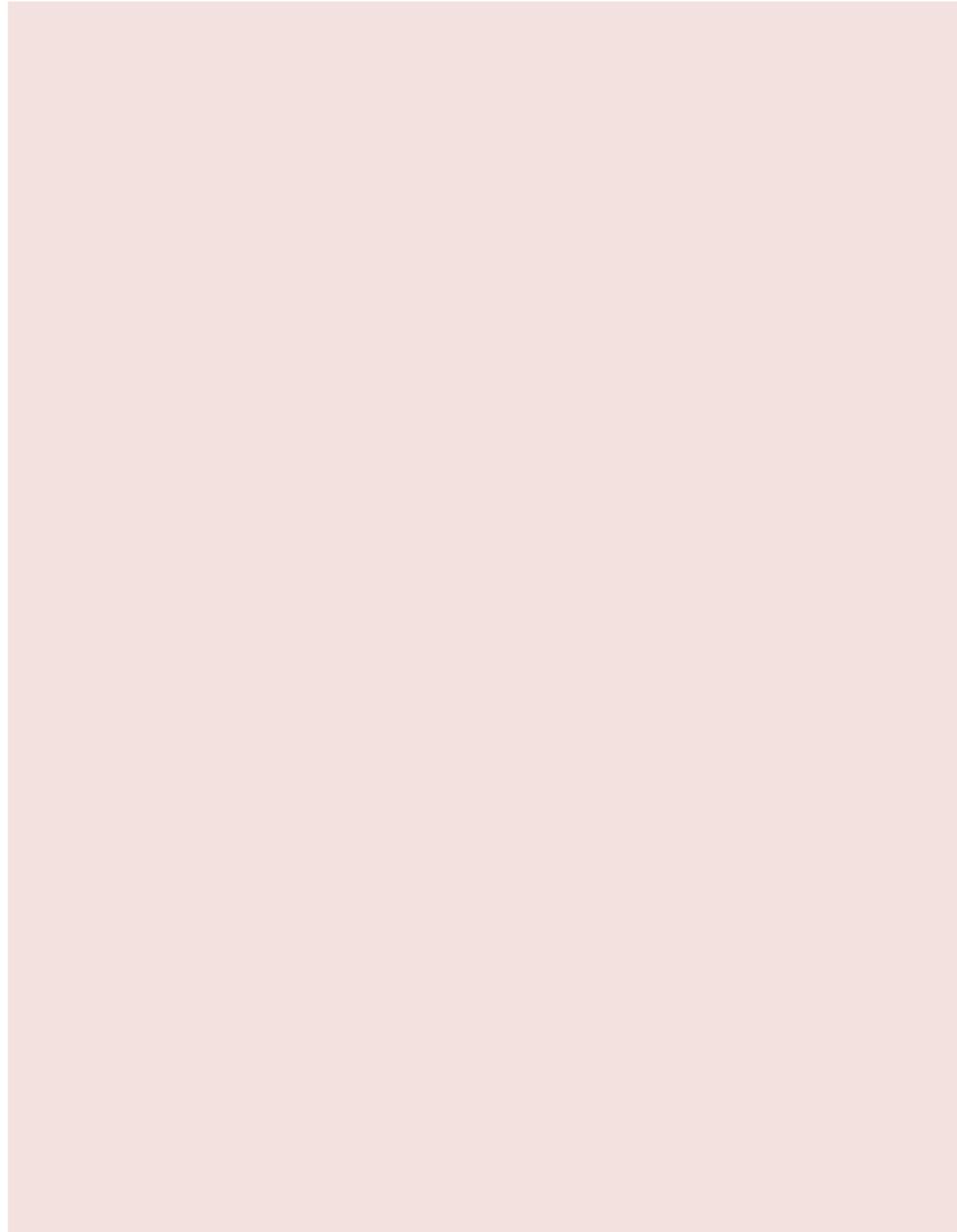
The Crystal World



The Crystal World



The Perfect World



The Real World



Massive Photon?

Hiding Symmetry

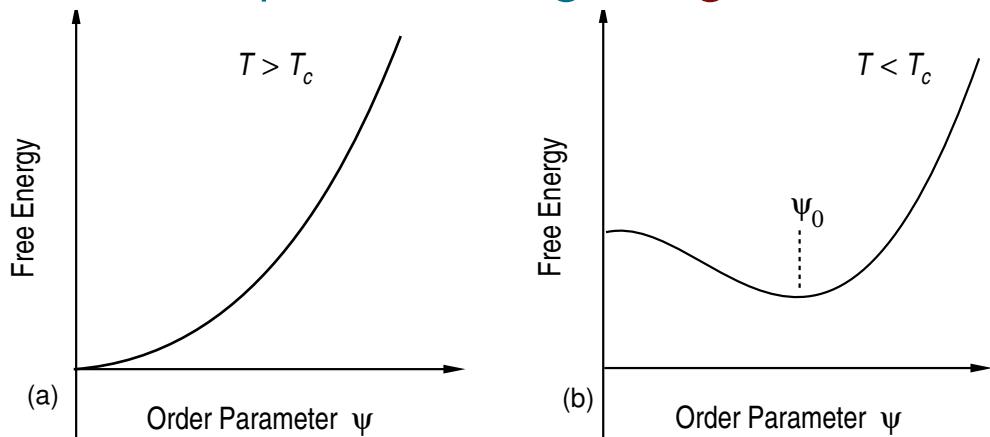
Recall [2] miracles of superconductivity:

- ▷ No resistance
- ▷ Meissner effect (exclusion of \mathbf{B})

Ginzburg–Landau Phenomenology
(not a theory from first principles)

normal, resistive charge carriers . . .

. . . + superconducting charge carriers



$\mathbf{B} = 0$:

$$G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha |\psi|^2 + \beta |\psi|^4$$

$$T > T_c : \quad \alpha > 0 \quad \langle |\psi|^2 \rangle_0 = 0$$

$$T < T_c : \quad \alpha < 0 \quad \langle |\psi|^2 \rangle_0 \neq 0$$

NONZERO MAGNETIC FIELD

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{8\pi} + \frac{1}{2m^*} \left| -i\hbar\nabla\psi - \frac{e^*}{c}\mathbf{A}\psi \right|^2$$

$$\left. \begin{array}{l} e^* = -2 \\ m^* \end{array} \right\} \text{of superconducting carriers}$$

Weak, slowly varying field

$$\psi \approx \psi_0 \neq 0, \nabla\psi \approx 0$$

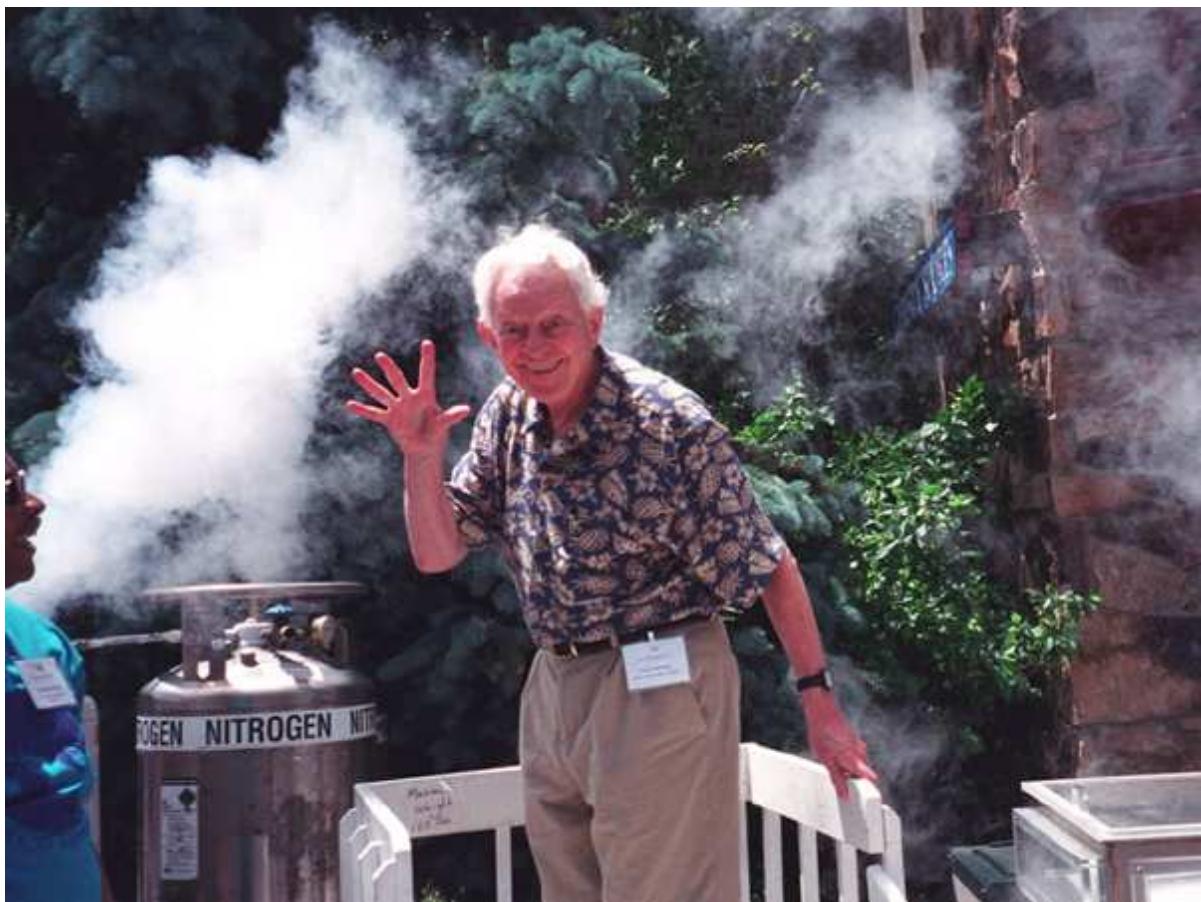
Variational analysis \implies

$$\boxed{\nabla^2\mathbf{A} - \frac{4\pi e^*}{m^* c^2} |\psi_0|^2 \mathbf{A} = 0}$$

wave equation of a *massive photon*

Photon—*gauge boson* — acquires mass
within superconductor

origin of Meissner effect



Meissner effect levitates Lederman, Snowmass 2001

Formulate electroweak theory

three crucial clues from experiment:

- ▷ Left-handed weak-isospin doublets,

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

and

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L ;$$

- ▷ Universal strength of the (charged-current) weak interactions;
- ▷ Idealization that neutrinos are massless.

First two clues suggest $SU(2)_L$ gauge symmetry

A theory of leptons

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad R \equiv e_R$$

weak hypercharges $Y_L = -1$, $Y_R = -2$

Gell-Mann–Nishijima connection, $Q = I_3 + \frac{1}{2}Y$

$SU(2)_L \otimes U(1)_Y$ gauge group \Rightarrow gauge fields:

- ★ weak isovector \vec{b}_μ , coupling g
- ★ weak isoscalar \mathcal{A}_μ , coupling $g'/2$

Field-strength tensors

$$F_{\mu\nu}^\ell = \partial_\nu b_\mu^\ell - \partial_\mu b_\nu^\ell + g \varepsilon_{jkl} b_\mu^j b_\nu^k , \text{ } SU(2)_L$$

and

$$f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu , \text{ } U(1)_Y$$

Interaction Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}} ,$$

with

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}^\ell F^{\ell\mu\nu} - \frac{1}{4}f_{\mu\nu}f^{\mu\nu},$$

and

$$\begin{aligned}\mathcal{L}_{\text{leptons}} &= \bar{R} i\gamma^\mu \left(\partial_\mu + i\frac{g'}{2} \mathcal{A}_\mu Y \right) R \\ &+ \bar{L} i\gamma^\mu \left(\partial_\mu + i\frac{g'}{2} \mathcal{A}_\mu Y + i\frac{g}{2} \vec{\tau} \cdot \vec{b}_\mu \right) L.\end{aligned}$$

Electron mass term

$$\mathcal{L}_e = -m_e (\bar{e}_R e_L + \bar{e}_L e_R) = -m_e \bar{e} e$$

would violate local gauge invariance Theory has four massless gauge bosons

$$\mathcal{A}_\mu \quad b_\mu^1 \quad b_\mu^2 \quad b_\mu^3$$

Nature has but one (γ)

Hiding EW Symmetry

*Higgs mechanism: relativistic generalization of
Ginzburg-Landau superconducting phase transition*

- ▷ Introduce a complex doublet of scalar fields

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad Y_\phi = +1$$

- ▷ Add to \mathcal{L} (gauge-invariant) terms for interaction and propagation of the scalars,

$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi),$$

where $\mathcal{D}_\mu = \partial_\mu + i \frac{g'}{2} \mathcal{A}_\mu Y + i \frac{g}{2} \vec{\tau} \cdot \vec{b}_\mu$ and

$$V(\phi^\dagger \phi) = \mu^2 (\phi^\dagger \phi) + |\lambda| (\phi^\dagger \phi)^2$$

- ▷ Add a Yukawa interaction

$$\mathcal{L}_{\text{Yukawa}} = -\zeta_e [\bar{R}(\phi^\dagger L) + (\bar{L}\phi)R]$$

- ▷ Arrange self-interactions so vacuum corresponds to a broken-symmetry solution: $\mu^2 < 0$
 Choose minimum energy (vacuum) state for vacuum expectation value

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \sqrt{-\mu^2/|\lambda|}$$

Hides (breaks) $SU(2)_L$ and $U(1)_Y$
 but preserves $U(1)_{\text{em}}$ invariance

Invariance under \mathcal{G} means $e^{i\alpha\mathcal{G}}\langle\phi\rangle_0 = \langle\phi\rangle_0$, so $\mathcal{G}\langle\phi\rangle_0 = 0$

$$\begin{aligned} \tau_1 \langle\phi\rangle_0 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!} \\ \tau_2 \langle\phi\rangle_0 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!} \\ \tau_3 \langle\phi\rangle_0 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!} \\ Y \langle\phi\rangle_0 &= Y_\phi \langle\phi\rangle_0 = +1 \langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!} \end{aligned}$$



Examine electric charge operator Q on the (electrically neutral) vacuum state

$$\begin{aligned}
 Q\langle\phi\rangle_0 &= \frac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 \\
 &= \frac{1}{2} \begin{pmatrix} Y_\phi + 1 & 0 \\ 0 & Y_\phi - 1 \end{pmatrix} \langle\phi\rangle_0 \\
 &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \\
 &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{unbroken!}
 \end{aligned}$$

Four original generators are broken

electric charge is not

▷ $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{em}}$ (will verify)

▷ Expect massless photon

▷ Expect gauge bosons corresponding to

$$\tau_1, \tau_2, \frac{1}{2}(\tau_3 - Y) \equiv K$$

to acquire masses

Expand about the vacuum state

Let $\phi = \begin{pmatrix} 0 \\ (v + \eta)/\sqrt{2} \end{pmatrix}$; in *unitary gauge*

$$\begin{aligned}\mathcal{L}_{\text{scalar}} &= \frac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) - \mu^2 \eta^2 \\ &\quad + \frac{v^2}{8}[g^2 |b_1 - ib_2|^2 + (g' \mathcal{A}_\mu - gb_\mu^3)^2] \\ &\quad + \text{interaction terms}\end{aligned}$$

Higgs boson η has acquired (mass)² $M_H^2 = -2\mu^2 > 0$

$$\frac{g^2 v^2}{8}(|W_\mu^+|^2 + |W_\mu^-|^2) \iff M_{W^\pm} = gv/2$$

Now define orthogonal combinations

$$Z_\mu = \frac{-g' \mathcal{A}_\mu + gb_\mu^3}{\sqrt{g^2 + g'^2}} \quad A_\mu = \frac{g \mathcal{A}_\mu + g' b_\mu^3}{\sqrt{g^2 + g'^2}}$$

$$M_{Z^0} = \sqrt{g^2 + g'^2} v/2 = M_W \sqrt{1 + g'^2/g^2}$$

A_μ remains massless

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= -\zeta_e \frac{(v + \eta)}{\sqrt{2}} (\bar{e}_R e_L + \bar{e}_L e_R) \\ &= -\frac{\zeta_e v}{\sqrt{2}} \bar{e} e - \frac{\zeta_e \eta}{\sqrt{2}} \bar{e} e\end{aligned}$$

electron acquires $m_e = \zeta_e v / \sqrt{2}$

Higgs coupling to electrons: m_e/v (\propto mass)

Desired particle content . . . + Higgs scalar

Values of couplings, electroweak scale v ?

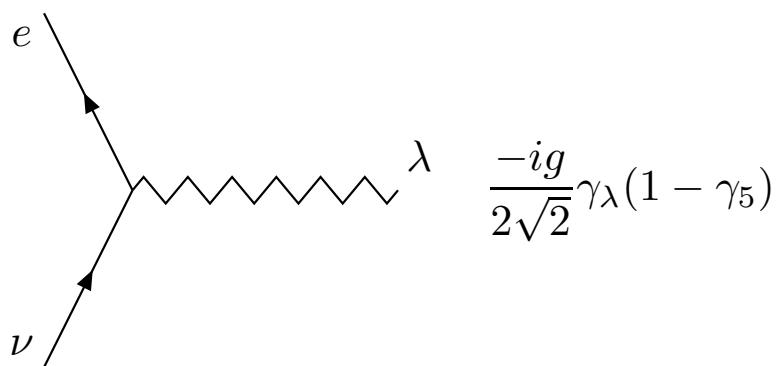
What about interactions?

Interactions . . .

$$\mathcal{L}_{W-\ell} = -\frac{g}{2\sqrt{2}} [\bar{\nu}\gamma^\mu(1-\gamma_5)eW_\mu^+ + \bar{e}\gamma^\mu(1-\gamma_5)\nu W_\mu^-]$$

+ similar terms for μ and τ

Feynman rule:



gauge-boson propagator:

$$W \quad \text{wavy line} \quad = \frac{-i(g_{\mu\nu} - k_\mu k_\nu/M_W^2)}{k^2 - M_W^2} .$$

Compute $\nu_\mu e \rightarrow \mu \nu_e$

$$\sigma(\nu_\mu e \rightarrow \mu \nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}$$

Reproduces 4-fermion result at low energies if

$$\begin{aligned} \frac{g^4}{16M_W^4} &= 2G_F^2 \\ \Rightarrow g^4 &= 32(G_F M_W^2)^2 = 64 \left(\frac{G_F M_W^2}{\sqrt{2}} \right)^2 \\ \Rightarrow \frac{g}{2\sqrt{2}} &= \left(\frac{G_F M_W^2}{\sqrt{2}} \right)^{\frac{1}{2}} \end{aligned}$$

Using $M_W = gv/2$, determine

$$v = (G_F \sqrt{2})^{-\frac{1}{2}} \approx 246 \text{ GeV}$$

the electroweak scale

$$\Rightarrow \langle \phi^0 \rangle_0 = (G_F \sqrt{8})^{-\frac{1}{2}} \approx 174 \text{ GeV}$$

W -propagator modifies HE behavior

$$\sigma(\nu_\mu e \rightarrow \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}$$

$$\lim_{E_\nu \rightarrow \infty} \sigma(\nu_\mu e \rightarrow \mu\nu_e) = \frac{g^4}{32\pi M_W^2} = \frac{G_F^2 M_W^2}{\sqrt{2}}$$

independent of energy!

partial-wave unitarity respected for

$$s < M_W^2 [\exp(\pi\sqrt{2}/G_F M_W^2) - 1]$$

W -boson properties

No prediction yet for M_W (haven't determined g)

Leptonic decay $W^- \rightarrow e^- \bar{\nu}_e$

$$e(p) \quad p \approx \left(\frac{M_W}{2}; \frac{M_W \sin \theta}{2}, 0, \frac{M_W \cos \theta}{2} \right)$$

$$\bar{\nu}_e(q) \quad q \approx \left(\frac{M_W}{2}; -\frac{M_W \sin \theta}{2}, 0, -\frac{M_W \cos \theta}{2} \right)$$

$$\mathcal{M} = -i \left(\frac{G_F M_W^2}{\sqrt{2}} \right)^{\frac{1}{2}} \bar{u}(e, p) \gamma_\mu (1 - \gamma_5) v(\nu, q) \varepsilon^\mu$$

$\varepsilon^\mu = (0; \hat{\varepsilon})$: W polarization vector in its rest frame

$$|\mathcal{M}|^2 = \frac{G_F M_W^2}{\sqrt{2}} \text{tr} [\not{q} (1 - \gamma_5) \not{p} (1 + \gamma_5) \not{\varepsilon}^* \not{p}] ;$$

$$\text{tr}[\dots] = [\varepsilon \cdot q \varepsilon^* \cdot p - \varepsilon \cdot \varepsilon^* q \cdot p + \varepsilon \cdot p \varepsilon^* \cdot q + i \epsilon_{\mu\nu\rho\sigma} \varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma]$$

decay rate is independent of W polarization; look first at longitudinal pol. $\varepsilon^\mu = (0; 0, 0, 1) = \varepsilon^{*\mu}$, eliminate $\epsilon_{\mu\nu\rho\sigma}$

$$|\mathcal{M}|^2 = \frac{4 G_F M_W^4}{\sqrt{2}} \sin^2 \theta$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2} \frac{\mathcal{S}_{12}}{M_W^3}$$

$$\mathcal{S}_{12} = \sqrt{[M_W^2 - (m_e + m_\nu)^2][M_W^2 - (m_e - m_\nu)^2]} = M_W^2$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{G_F M_W^3}{16\pi^2 \sqrt{2}} \sin^2 \theta$$

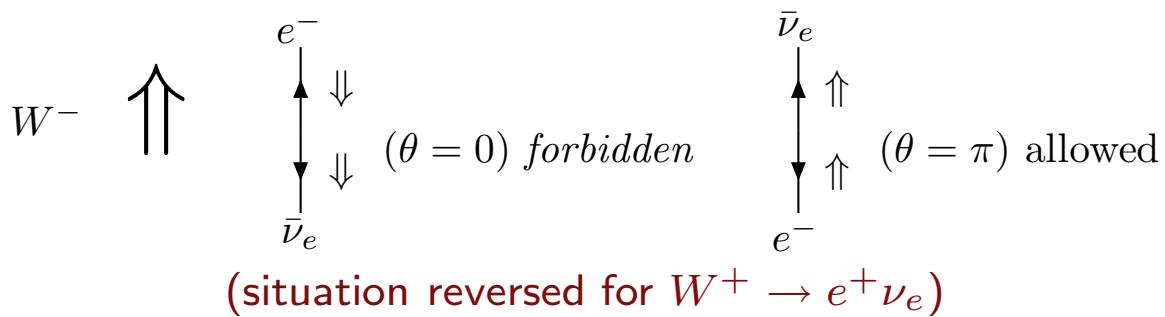
and

$$\boxed{\Gamma(W \rightarrow e\nu) = \frac{G_F M_W^3}{6\pi \sqrt{2}}}$$

Other helicities: $\varepsilon_{\pm 1}^\mu = (0; -1, \mp i, 0)/\sqrt{2}$

$$\frac{d\Gamma_{\pm 1}}{d\Omega} = \frac{G_F M_W^3}{32\pi^2 \sqrt{2}} (1 \mp \cos \theta)^2$$

Extinctions at $\cos \theta = \pm 1$ are consequences of angular momentum conservation:



e^+ follows polarization direction of W^+

e^- avoids polarization direction of W^-

important for discovery of W^\pm in $\bar{p}p$ ($\bar{q}q$) C violation

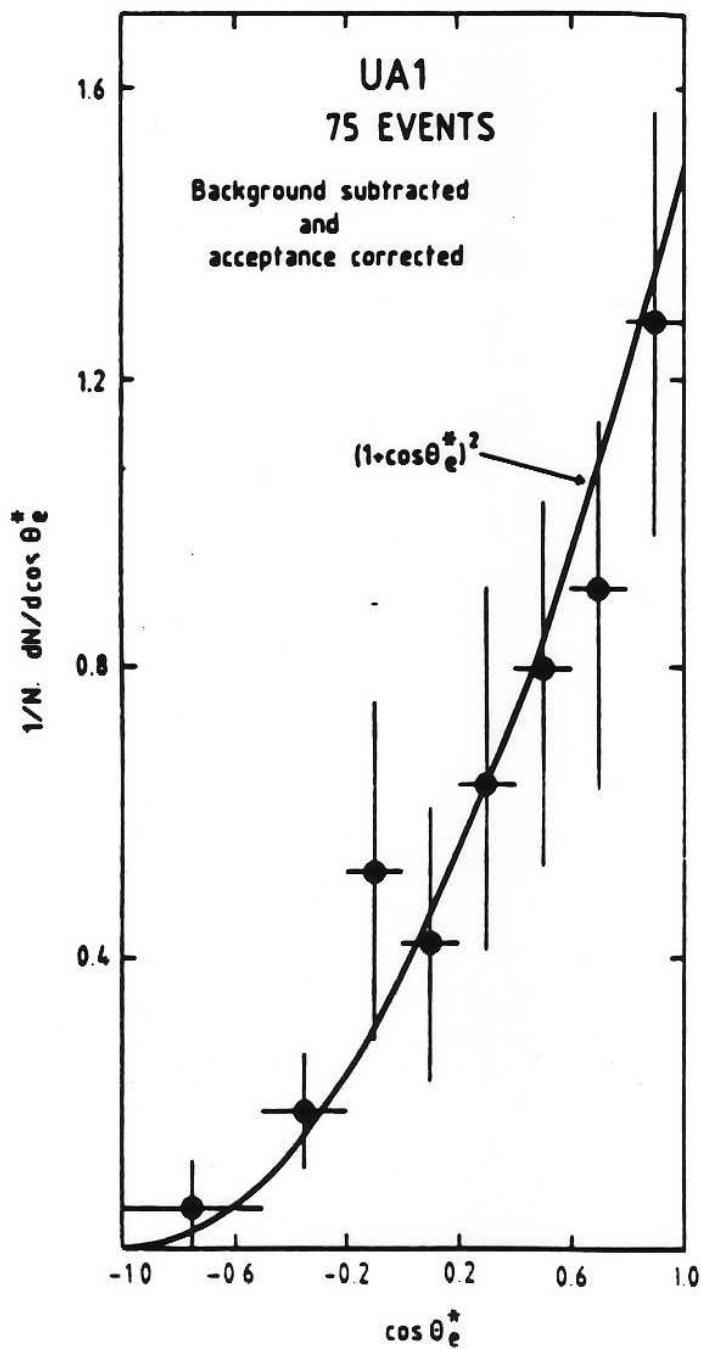


Fig. 2. The W decay angular distribution of the emission angle θ^* of the electron (positron) with respect to the proton (anti-proton) direction in the rest frame of the W. Only those events for which the lepton charge and the decay kinematics are well determined have been used. The curve shows the ($V - A$) expectation of $(1 + \cos\theta^*)^2$.

Interactions . . .

$$\mathcal{L}_{A-\ell} = \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e} \gamma^\mu e A_\mu$$

. . . vector interaction; $\Rightarrow A_\mu$ as γ , provided

$$gg' / \sqrt{g^2 + g'^2} \equiv e$$

Define $g' = g \tan \theta_W$ θ_W : weak mixing angle

$$g = e / \sin \theta_W \geq e$$

$$g' = e / \cos \theta_W \geq e$$

$$Z_\mu = b_\mu^3 \cos \theta_W - \mathcal{A}_\mu \sin \theta_W \quad A_\mu = \mathcal{A}_\mu \cos \theta_W + b_\mu^3 \sin \theta_W$$

$$\mathcal{L}_{Z-\nu} = \frac{-g}{4 \cos \theta_W} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu Z_\mu$$

$$\mathcal{L}_{Z-e} = \frac{-g}{4 \cos \theta_W} \bar{e} [L_e \gamma^\mu (1 - \gamma_5) + R_e \gamma^\mu (1 + \gamma_5)] e Z_\mu$$

$$L_e = 2 \sin^2 \theta_W - 1 = 2x_W + \tau_3$$

$$R_e = 2 \sin^2 \theta_W = 2x_W$$

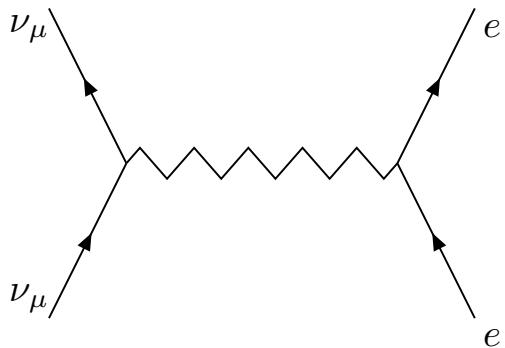
Z -boson properties

Decay calculation analogous to W^\pm

$$\begin{aligned}\Gamma(Z \rightarrow \nu\bar{\nu}) &= \frac{G_F M_Z^3}{12\pi\sqrt{2}} \\ \Gamma(Z \rightarrow e^+e^-) &= \Gamma(Z \rightarrow \nu\bar{\nu}) [L_e^2 + R_e^2]\end{aligned}$$

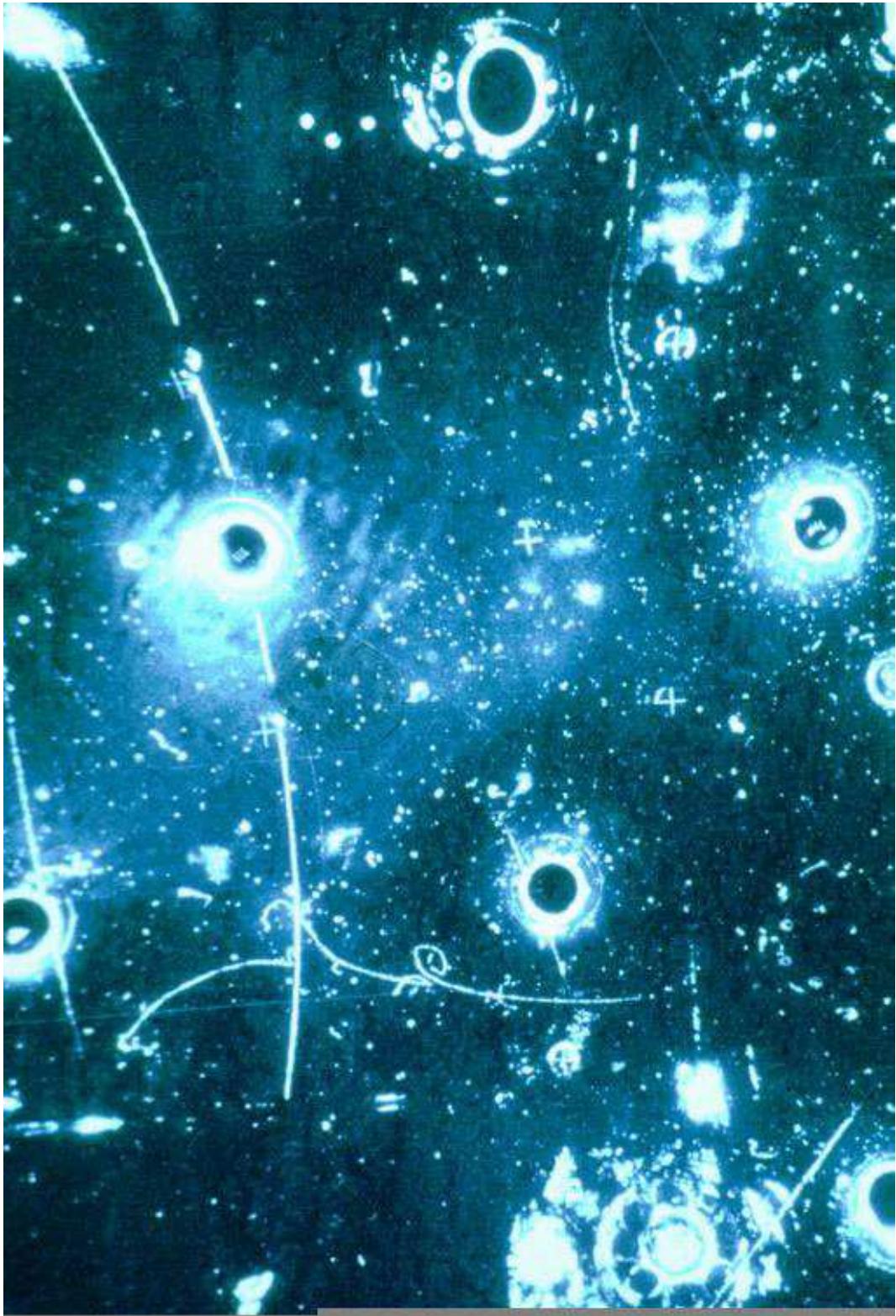
Neutral-current interactions

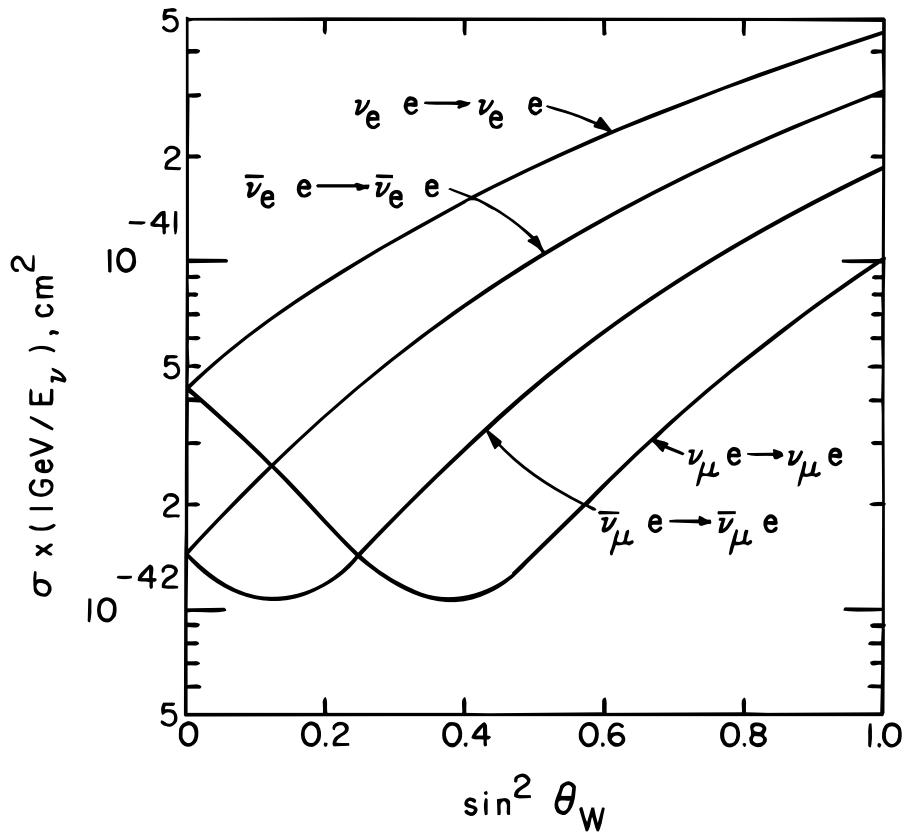
New νe reaction, not present in $V - A$



$$\begin{aligned}\sigma(\nu_\mu e \rightarrow \nu_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi} [L_e^2 + R_e^2/3] \\ \sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi} [L_e^2/3 + R_e^2] \\ \sigma(\nu_e e \rightarrow \nu_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi} [(L_e + 2)^2 + R_e^2/3] \\ \sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi} [(L_e + 2)^2/3 + R_e^2]\end{aligned}$$

Gargamelle $\nu_\mu e$ Event





“Model-independent” analysis

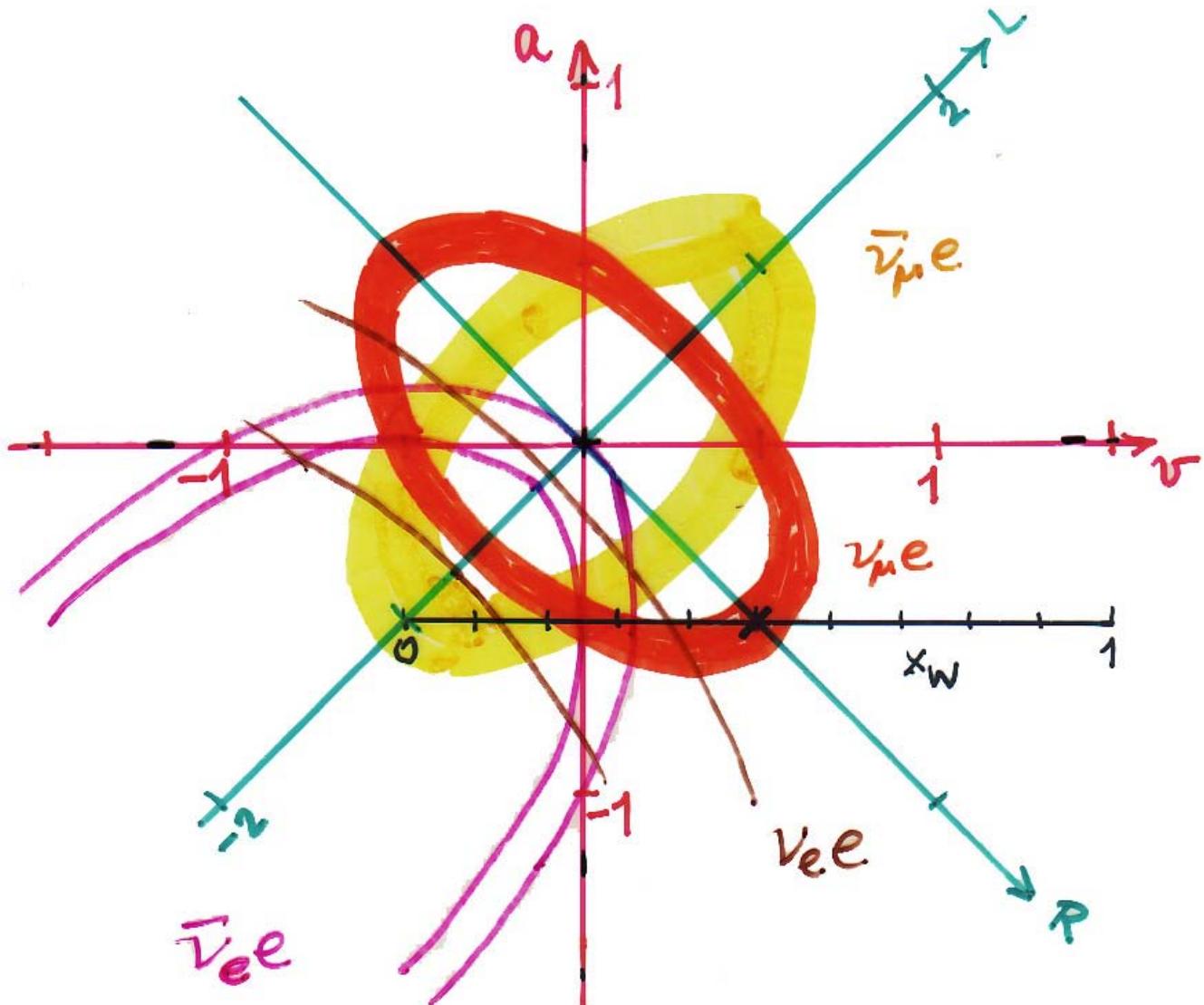
Measure all cross sections to determine chiral couplings L_e and R_e or traditional vector and axial couplings v and a

$$a = \frac{1}{2}(L_e - R_e) \quad v = \frac{1}{2}(L_e + R_e)$$

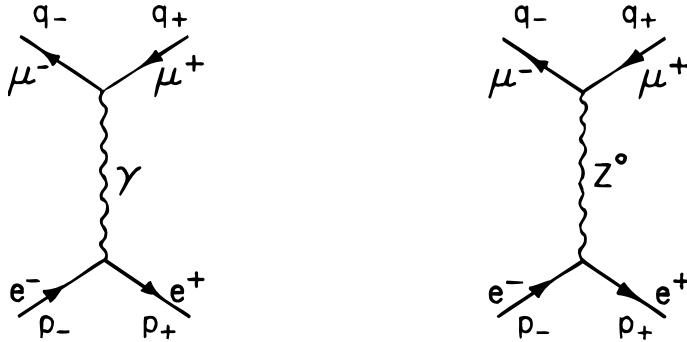
$$L_e = v + a \quad R_e = v - a$$

model-independent in V, A framework

Neutrino-electron scattering



Twofold ambiguity remains even after measuring all four cross sections: same cross sections result if we interchange $R_e \leftrightarrow -R_e$ ($v \leftrightarrow a$) Consider $e^+e^- \rightarrow \mu^+\mu^-$



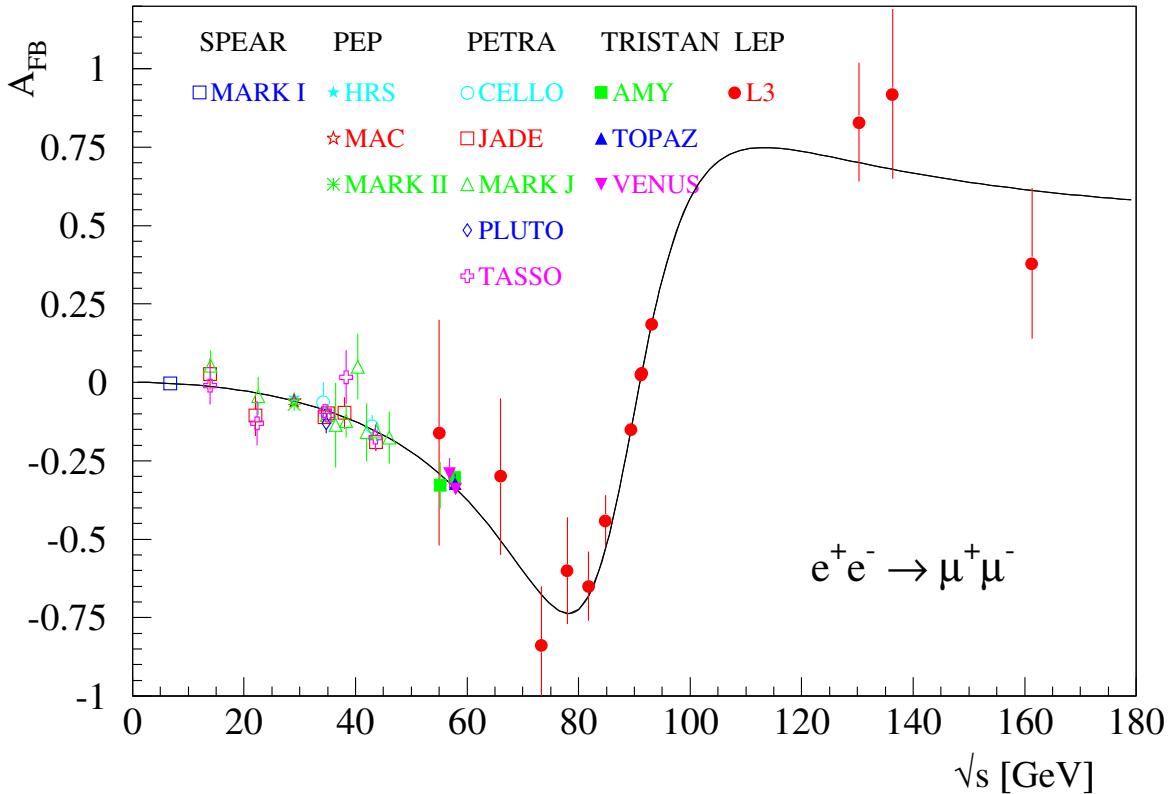
$$\begin{aligned} \mathcal{M} = & -ie^2 \bar{u}(\mu, q_-) \gamma_\lambda Q_\mu v(\mu, q_+) \frac{g^{\lambda\nu}}{s} \bar{v}(e, p_+) \gamma_\nu u(e.p_-) \\ & + \frac{i}{2} \left(\frac{G_F M_Z^2}{\sqrt{2}} \right) \bar{u}(\mu, q_-) \gamma_\lambda [R_\mu (1 + \gamma_5) + L_\mu (1 - \gamma_5)] v(\mu, q_+) \\ & \times \frac{g^{\lambda\nu}}{s - M_Z^2} \bar{v}(e, p_+) \gamma_\nu [R_e (1 + \gamma_5) + L_e (1 - \gamma_5)] u(e.p_-) \end{aligned}$$

muon charge $Q_\mu = -1$

$$\begin{aligned} \frac{d\sigma}{dz} = & \frac{\pi \alpha^2 Q_\mu^2}{2s} (1 + z^2) \\ & - \frac{\alpha Q_\mu G_F M_Z^2 (s - M_Z^2)}{8\sqrt{2}[(s - M_Z^2)^2 + M_Z^2 \Gamma^2]} \\ & \times [(R_e + L_e)(R_\mu + L_\mu)(1 + z^2) + 2(R_e - L_e)(R_\mu - L_\mu)z] \\ & + \frac{G_F^2 M_Z^4 s}{64\pi[(s - M_Z^2)^2 + M_Z^2 \Gamma^2]} \\ & \times [(R_e^2 + L_e^2)(R_\mu^2 + L_\mu^2)(1 + z^2) + 2(R_e^2 - L_e^2)(R_\mu^2 - L_\mu^2)z] \end{aligned}$$

$$\text{F-B asymmetry } A \equiv \frac{\int_0^1 dz d\sigma/dz - \int_{-1}^0 dz d\sigma/dz}{\int_{-1}^1 dz d\sigma/dz}$$

$$\begin{aligned}\lim_{s/M_Z^2 \ll 1} A &= \frac{3G_F s}{16\pi\alpha Q_\mu \sqrt{2}} (R_e - L_e)(R_\mu - L_\mu) \\ &\approx -6.7 \times 10^{-5} \left(\frac{s}{1 \text{ GeV}^2} \right) (R_e - L_e)(R_\mu - L_\mu) \\ &= -3G_F s a^2 / 4\pi\alpha\sqrt{2}\end{aligned}$$

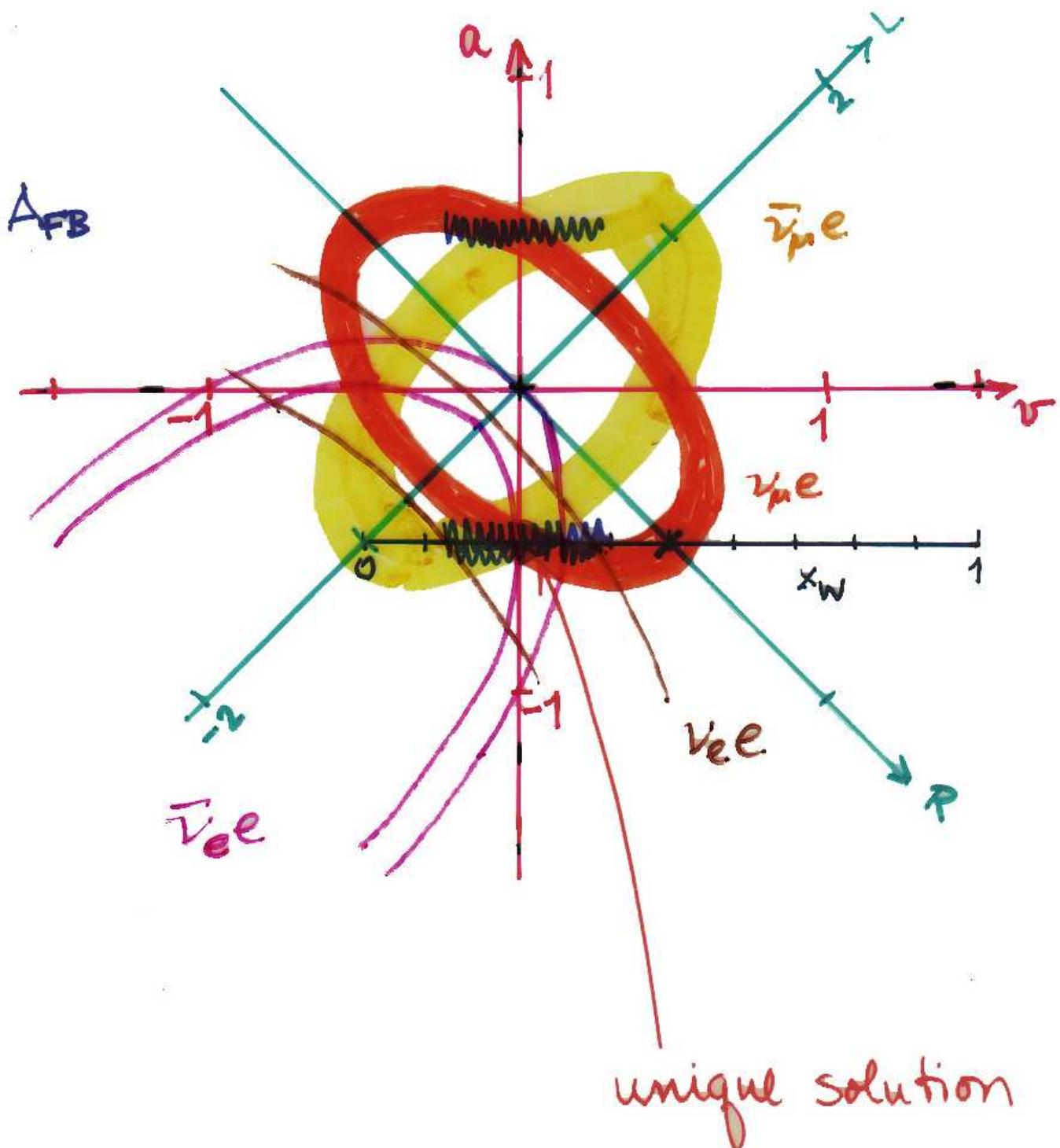


J. Mnich *Phys. Rep.* **271**, 181-266 (1996)

Measuring A resolves ambiguity

Validate EW theory, measure $\sin^2 \theta_W$

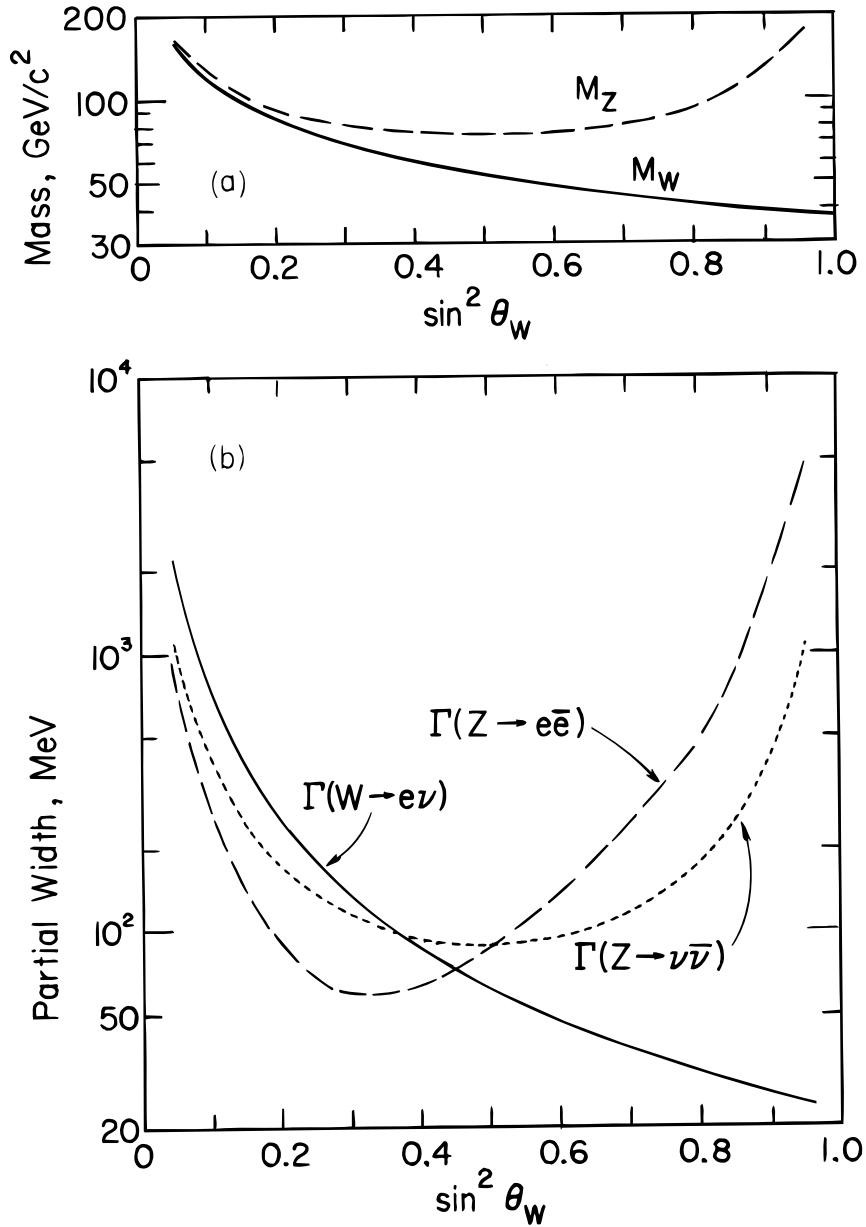
Neutrino-electron scattering



With a measurement of $\sin^2 \theta_W$, predict

$$M_W^2 = g^2 v^2 / 4 = e^2 / 4G_F \sqrt{2} \sin^2 \theta_W \approx (37.3 \text{ GeV}/c^2)^2 / \sin^2 \theta_W$$

$$M_Z^2 = M_W^2 / \cos^2 \theta_W$$



EW interactions of quarks

- ▷ Left-handed doublet

$$I_3 \quad Q \quad Y = 2(Q - I_3)$$

$$\mathsf{L}_q = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{matrix} \frac{1}{2} \\ -\frac{1}{2} \end{matrix} \quad \begin{matrix} +\frac{2}{3} \\ -\frac{1}{3} \end{matrix} \quad \frac{1}{3}$$

- ▷ two right-handed singlets

$$I_3 \quad Q \quad Y = 2(Q - I_3)$$

$$\mathsf{R}_u = u_R \quad 0 \quad +\frac{2}{3} \quad +\frac{4}{3}$$

$$\mathsf{R}_d = d_R \quad 0 \quad -\frac{1}{3} \quad -\frac{2}{3}$$

- ▷ CC interaction

$$\mathcal{L}_{W-q} = \frac{-g}{2\sqrt{2}} [\bar{u}_e \gamma^\mu (1 - \gamma_5) d W_\mu^+ + \bar{d} \gamma^\mu (1 - \gamma_5) u W_\mu^-]$$

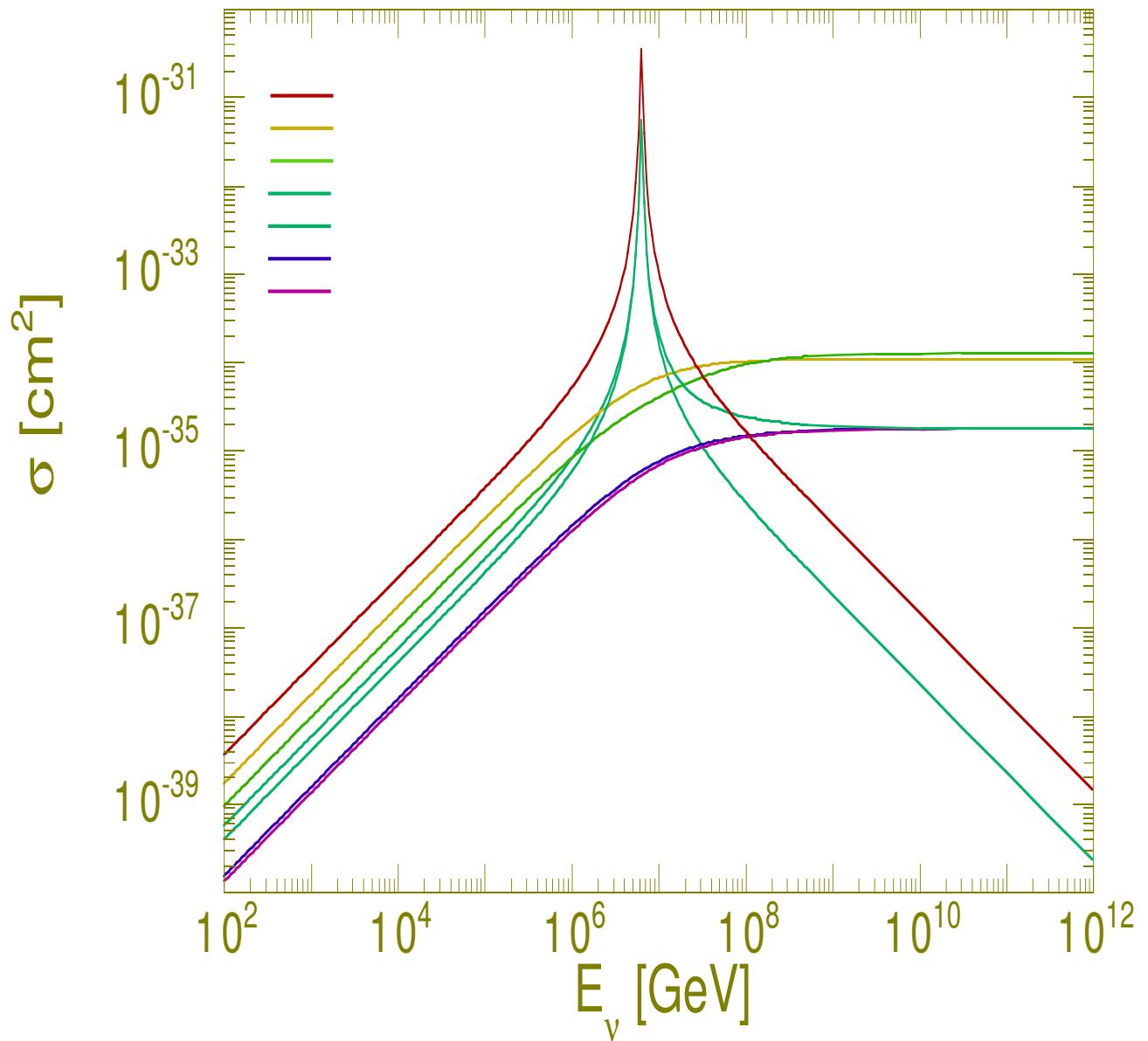
identical in form to $\mathcal{L}_{W-\ell}$: universality \Leftrightarrow weak isospin

- ▷ NC interaction

$$\mathcal{L}_{Z-q} = \frac{-g}{4 \cos \theta_W} \sum_{i=u,d} \bar{q}_i \gamma^\mu [L_i(1 - \gamma_5) + R_i(1 + \gamma_5)] q_i Z_\mu$$

$$L_i = \tau_3 - 2Q_i \sin^2 \theta_W \quad R_i = -2Q_i \sin^2 \theta_W$$

equivalent in form (not numbers) to $\mathcal{L}_{Z-\ell}$



At low energies: $\sigma(\bar{\nu}_e e \rightarrow \text{hadrons}) > \sigma(\nu_\mu e \rightarrow \mu\nu_e) >$
 $\sigma(\nu_e e \rightarrow \nu_e e) > \sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_\mu \mu) > \sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) >$
 $\sigma(\nu_\mu e \rightarrow \nu_\mu e) > \sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e)$

Trouble in Paradise

Universal $u \leftrightarrow d$, $\nu_e \leftrightarrow e$ not quite right

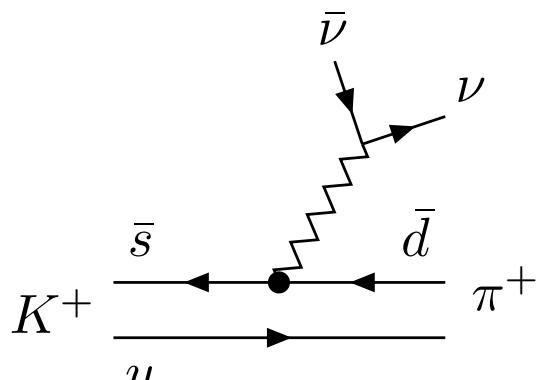
$$\text{Good: } \begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow \text{Better: } \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L$$

$$d_\theta \equiv d \cos \theta_C + s \sin \theta_C \quad \cos \theta_C = 0.9736 \pm 0.0010$$

“Cabibbo-rotated” doublet perfects CC interaction (up to small third-generation effects) but \Rightarrow serious trouble for NC

$$\begin{aligned} \mathcal{L}_{Z-q} = & \frac{-g}{4 \cos \theta_W} Z_\mu \{ \bar{u} \gamma^\mu [L_u(1 - \gamma_5) + R_u(1 + \gamma_5)] u \\ & + \bar{d} \gamma^\mu [L_d(1 - \gamma_5) + R_d(1 + \gamma_5)] d \cos^2 \theta_C \\ & + \bar{s} \gamma^\mu [L_d(1 - \gamma_5) + R_d(1 + \gamma_5)] s \sin^2 \theta_C \\ & + \bar{d} \gamma^\mu [L_d(1 - \gamma_5) + R_d(1 + \gamma_5)] s \sin \theta_C \cos \theta_C \\ & + \bar{s} \gamma^\mu [L_d(1 - \gamma_5) + R_d(1 + \gamma_5)] d \sin \theta_C \cos \theta_C \} \end{aligned}$$

Strangeness-changing NC interactions highly suppressed!



(SM: 0.8 ± 0.3)

BNL E-787/E-949 has three $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ candidates, with $\mathcal{B}(K^+ \rightarrow \pi^+ \nu\bar{\nu}) = 1.47^{+1.30}_{-0.89} \times 10^{-10}$

Phys. Rev. Lett. **93**, 031801 (2004)

Glashow–Iliopoulos–Maiani

two left-handed doublets

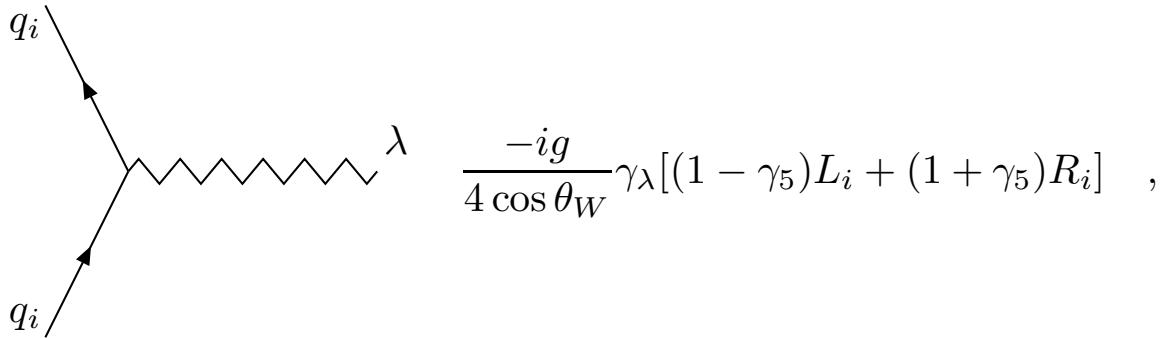
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \quad \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L$$

$$(s_\theta = s \cos \theta_C - d \sin \theta_C)$$

+ right-handed singlets, $e_R, \mu_R, u_R, d_R, c_R, s_R$

Required new charmed quark, c

Cross terms vanish in \mathcal{L}_{Z-q} ,



$$L_i = \tau_3 - 2Q_i \sin^2 \theta_W \quad R_i = -2Q_i \sin^2 \theta_W$$

flavor-diagonal interaction!

Straightforward generalization to n quark doublets

$$\mathcal{L}_{W-q} = \frac{-g}{2\sqrt{2}} [\bar{\Psi} \gamma^\mu (1 - \gamma_5) \mathcal{O} \Psi W_\mu^+ + \text{h.c.}]$$

composite $\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ d \\ s \\ \vdots \end{pmatrix}$

flavor structure $\mathcal{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}$

U : unitary quark mixing matrix

Weak-isospin part: $\mathcal{L}_{Z-q}^{\text{iso}} = \frac{-g}{4 \cos \theta_W} \bar{\Psi} \gamma^\mu (1 - \gamma_5) [\mathcal{O}, \mathcal{O}^\dagger] \Psi$

Since $[\mathcal{O}, \mathcal{O}^\dagger] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \propto \tau_3$

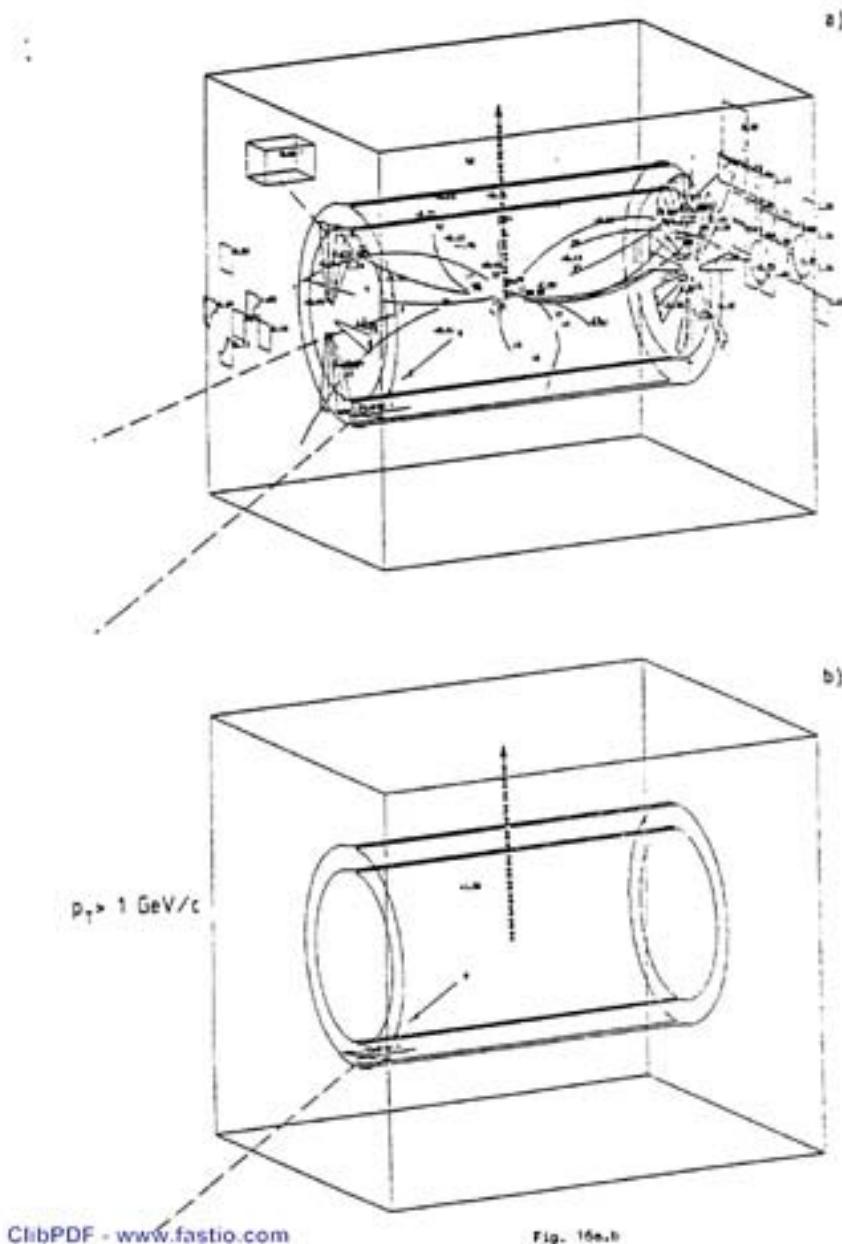
\Rightarrow NC interaction is flavor-diagonal

General $n \times n$ quark-mixing matrix U :

$n(n-1)/2$ real \angle , $(n-1)(n-2)/2$ complex phases

3×3 (Cabibbo–Kobayashi–Maskawa): $3 \angle + 1$ phase
 \Rightarrow CP violation

568 Intermediate Vector Bosons W^+ , W^- , and Z^0



ClipPDF - www.fastio.com

Fig. 16a,b

UA1

Qualitative successes of $SU(2)_L \otimes U(1)_Y$ theory:

- ▷ neutral-current interactions
- ▷ necessity of charm
- ▷ existence and properties of W^\pm and Z^0

Decade of precision tests EW (one-per-mille)

M_Z	$91\,187.6 \pm 2.1$ MeV/c ²
Γ_Z	2495.2 ± 2.3 MeV
$\sigma_{\text{hadronic}}^0$	41.541 ± 0.037 nb
Γ_{hadronic}	1744.4 ± 2.0 MeV
Γ_{leptonic}	83.984 ± 0.086 MeV
$\Gamma_{\text{invisible}}$	499.0 ± 1.5 MeV

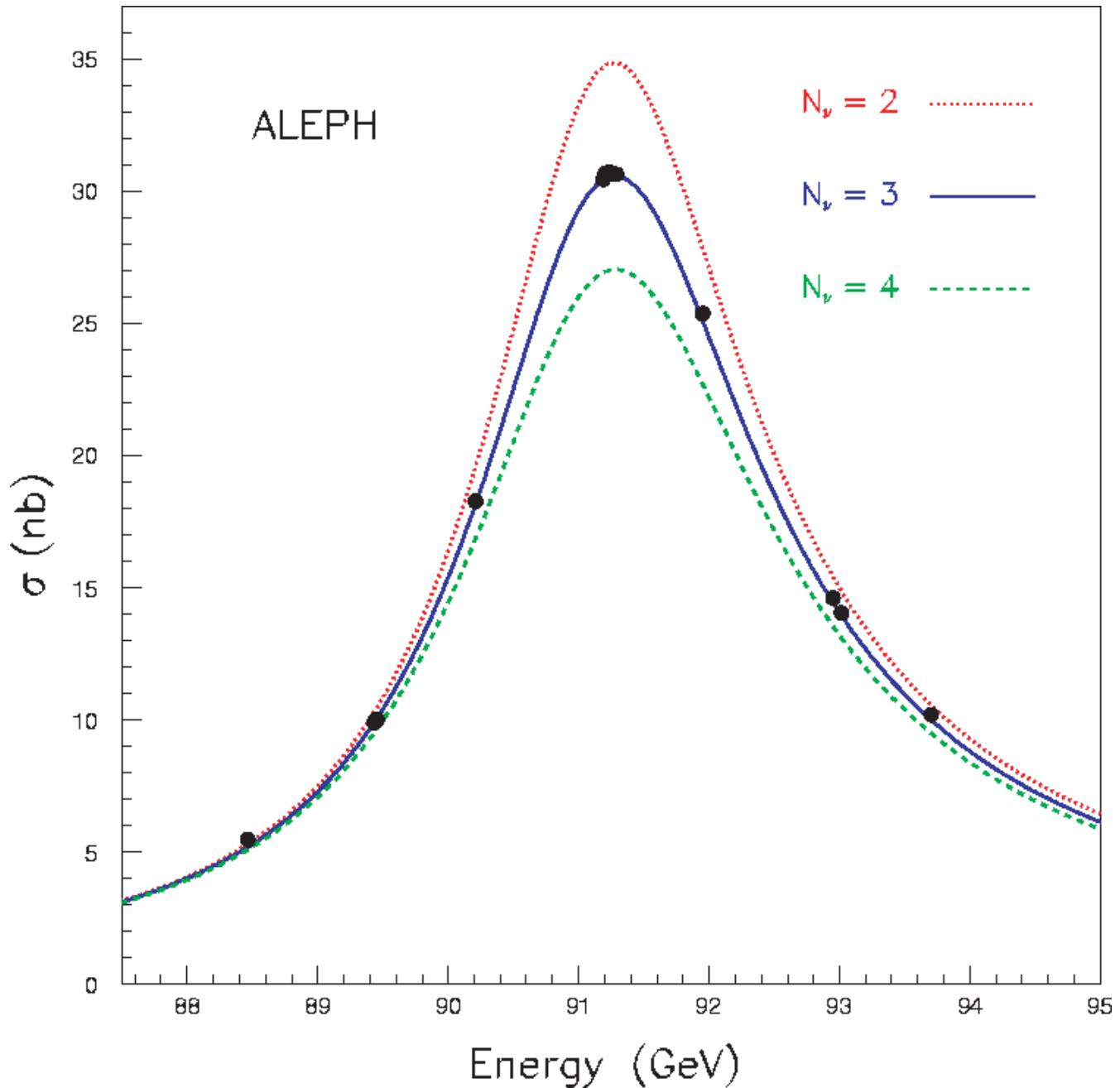
where $\Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}}$

light neutrinos $N_\nu = \Gamma_{\text{invisible}}/\Gamma^{\text{SM}}(Z \rightarrow \nu_i \bar{\nu}_i)$

Current value: $N_\nu = 2.994 \pm 0.012$

. . . excellent agreement with ν_e , ν_μ , and ν_τ

Three light neutrinos



The top quark must exist

- ▷ Two families

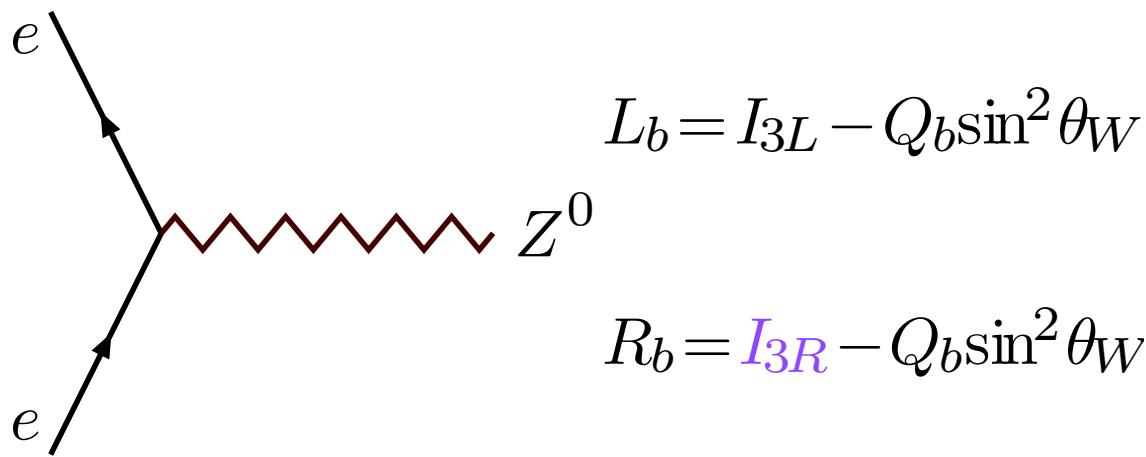
$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L$$

don't account for CP violation. Need a third family . . . or another answer.

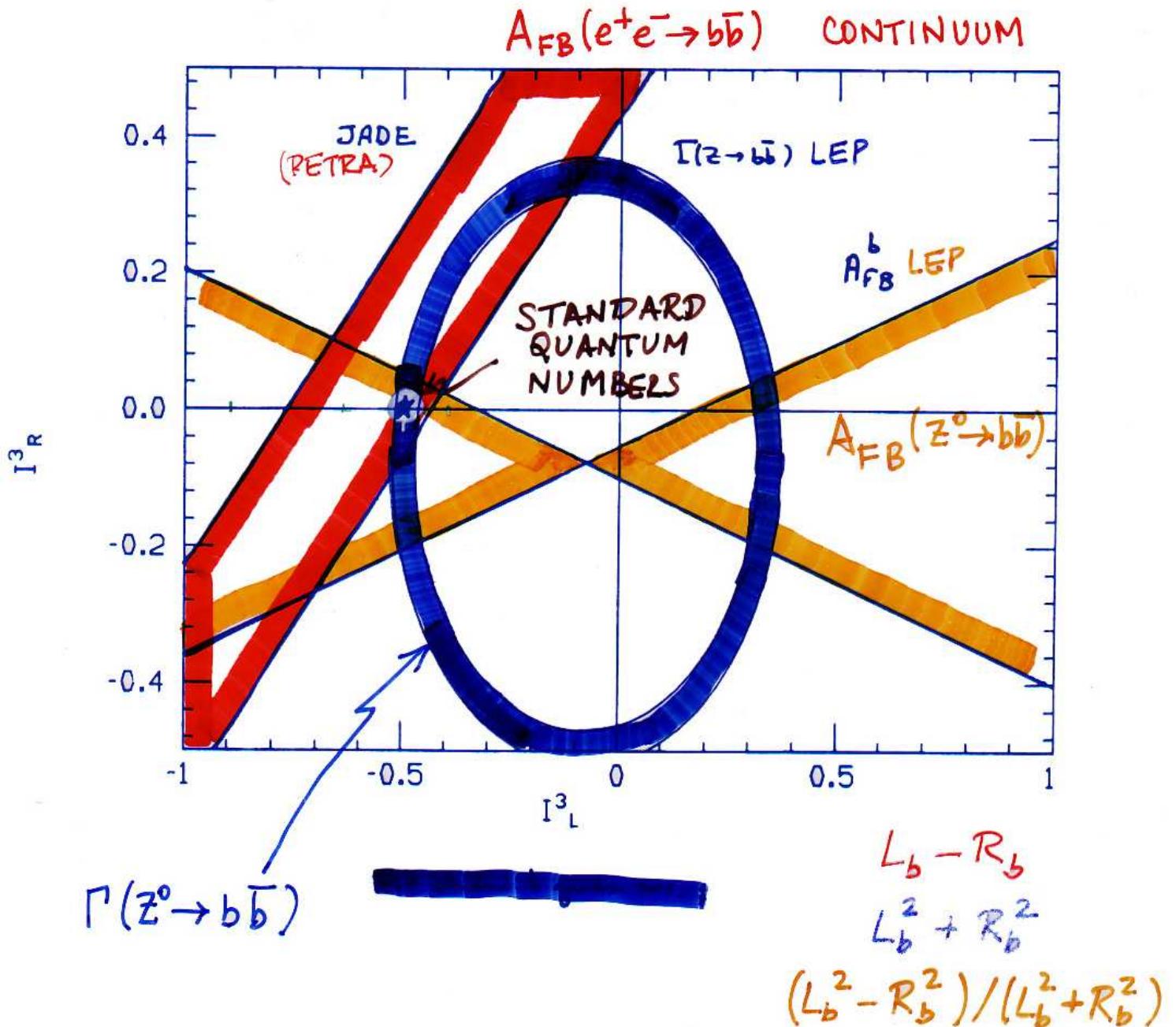
Given the existence of b , (τ)

- ▷ top is needed for an anomaly-free EW theory
- ▷ absence of FCNC in b decay ($b \not\rightarrow s\ell^+\ell^-$, etc.)
- ▷ b has weak isospin $I_{3L} = -\frac{1}{2}$; needs partner

$$\begin{pmatrix} t \\ b \end{pmatrix}_L$$



Measure $I_{3L}^{(b)} = -0.490^{+0.015}_{-0.012}$ $I_{3R}^{(b)} = -0.028 \pm 0.056$



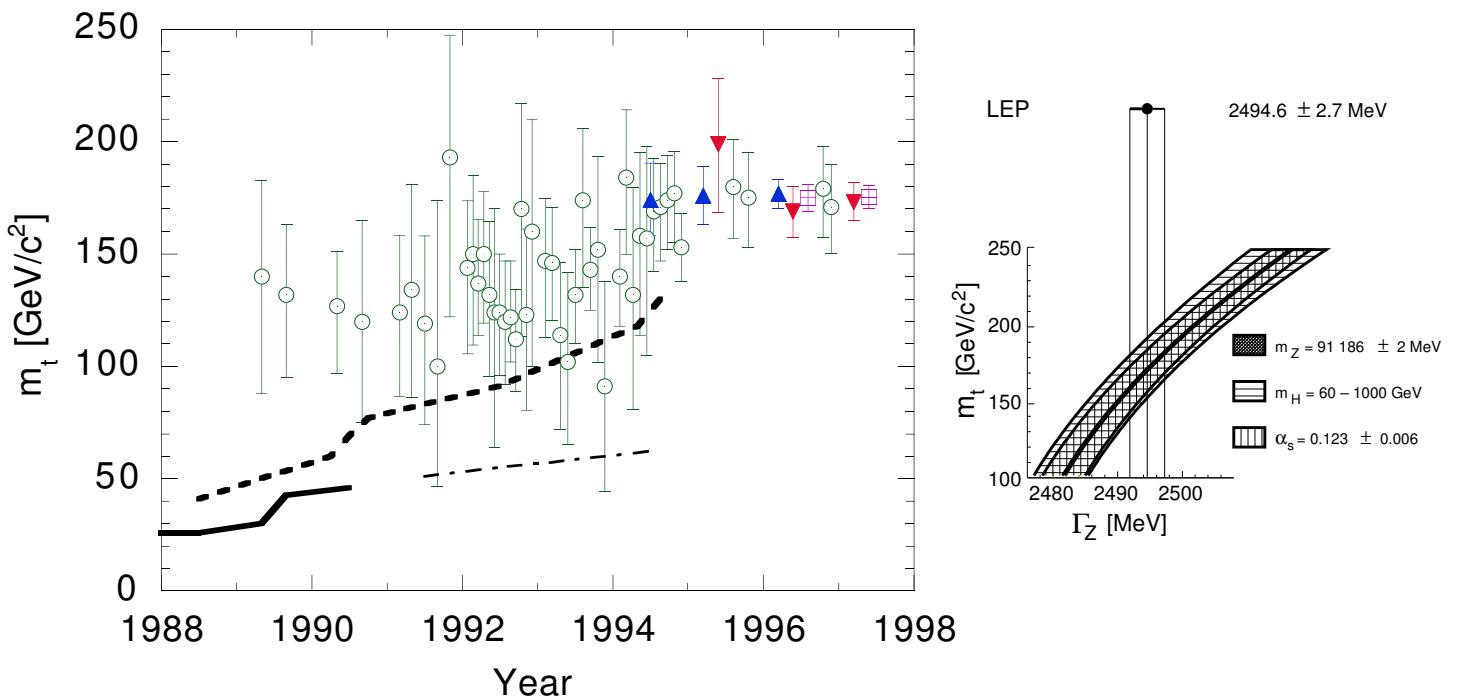
Needed: top with $I_{3L} = +\frac{1}{2}$

D. Schaile & P. Zerwas, *Phys. Rev. D45*, 3262 (1992)

Global fits . . .

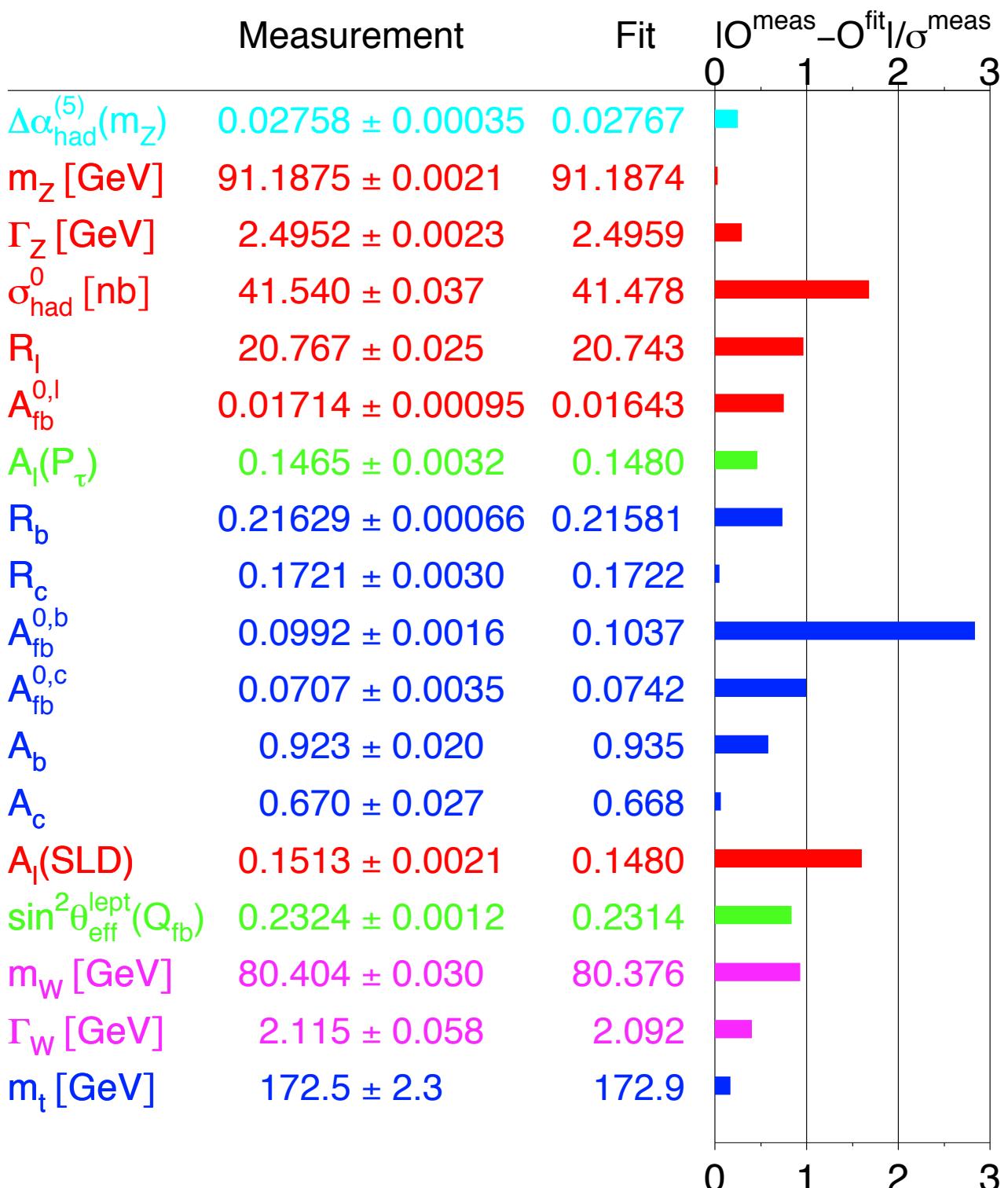
to precision EW measurements:

- ▷ precision improves with time
- ▷ calculations improve with time



11.94, LEPEWWG: $m_t = 178 \pm 11^{+18}_{-19}$ GeV/c²

Direct measurements: $m_t = 172.5 \pm 2.3$ GeV/c²



LEP Electroweak Working Group, Winter 2006

The Origins of Mass

(masses of nuclei “understood”)

$p, [\pi], \rho$ understood: QCD

confinement energy is the source

“Mass without mass”

Wilczek, *Phys. Today* (November 1999)

We understand the visible mass of the Universe
... without the Higgs mechanism

W, Z electroweak symmetry breaking

$$M_W^2 = \frac{1}{2}g^2 v^2 = \pi\alpha/G_F \sqrt{2} \sin^2 \theta_W$$

$$M_Z^2 = M_W^2 / \cos^2 \theta_W$$

q, ℓ^\mp EWSB + Yukawa couplings

ν_ℓ EWSB + Yukawa couplings; new physics?

All fermion masses \Leftrightarrow physics beyond standard model

H ?? fifth force ??

The vacuum energy problem

Higgs potential $V(\varphi^\dagger \varphi) = \mu^2 (\varphi^\dagger \varphi) + |\lambda| (\varphi^\dagger \varphi)^2$

At the minimum,

$$V(\langle \varphi^\dagger \varphi \rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda| v^4}{4} < 0.$$

Identify $M_H^2 = -2\mu^2$

contributes field-independent vacuum energy density

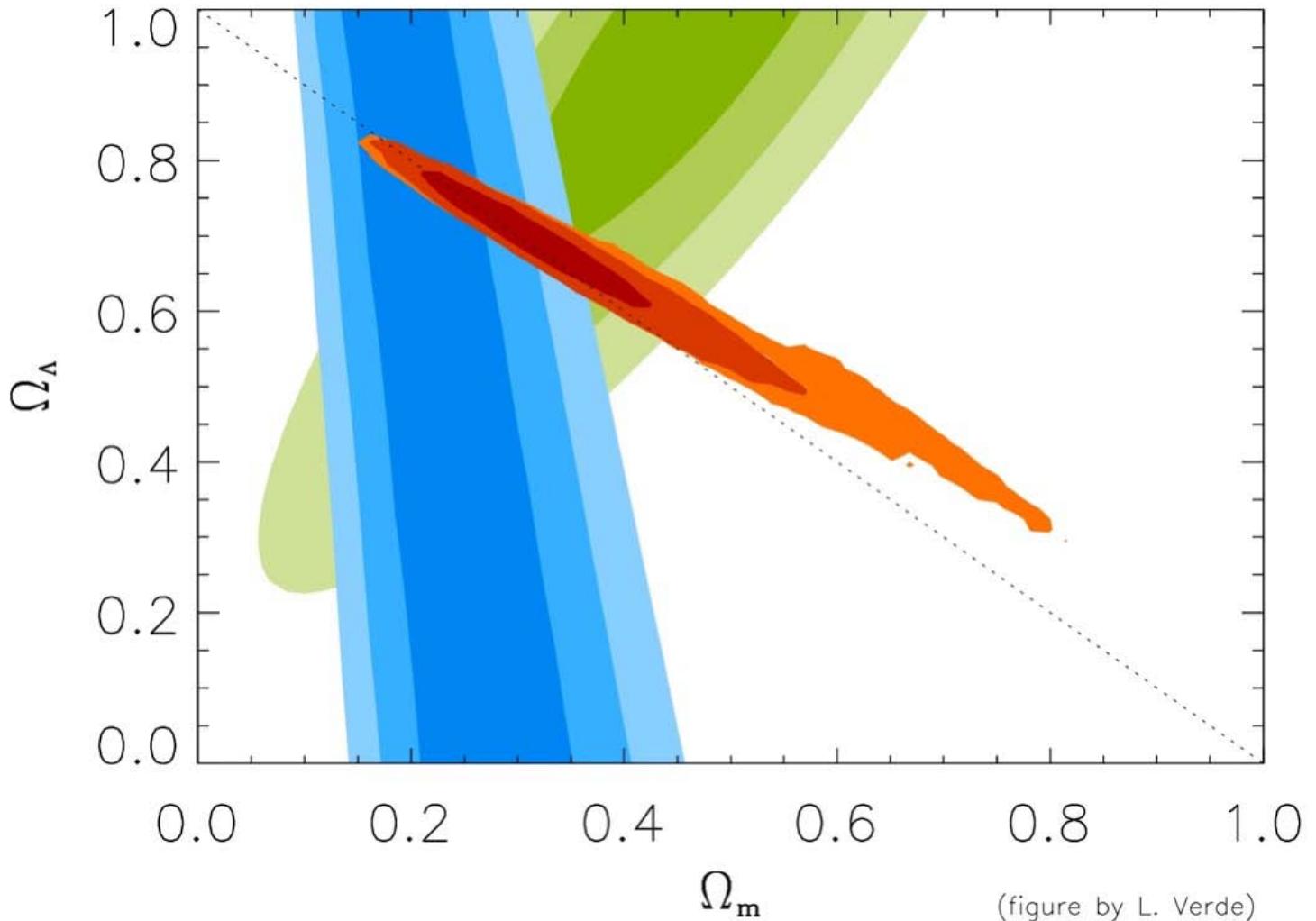
$$\varrho_H \equiv \frac{M_H^2 v^2}{8}$$

Adding vacuum energy density ϱ_{vac} \Leftrightarrow adding cosmological constant Λ to Einstein's equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu}$$

$$\Lambda = \frac{8\pi G_N}{c^4} \varrho_{\text{vac}}$$

observed vacuum energy density $\varrho_{\text{vac}} \lesssim 10^{-46} \text{ GeV}^4$



But $M_H \gtrsim 114 \text{ GeV}/c^2 \Rightarrow$

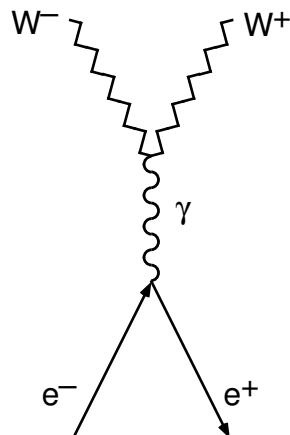
$$\varrho_H \gtrsim 10^8 \text{ GeV}^4$$

MISMATCH BY 54 ORDERS OF MAGNITUDE

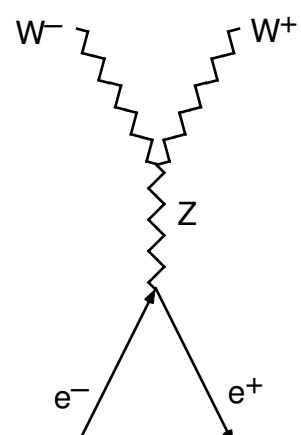
Why a Higgs Boson Must Exist

▷ Role in canceling high-energy divergences

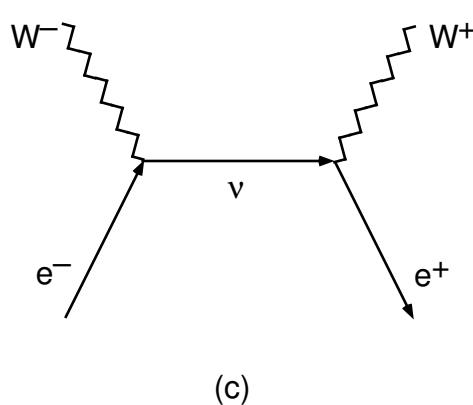
S-matrix analysis of $e^+e^- \rightarrow W^+W^-$



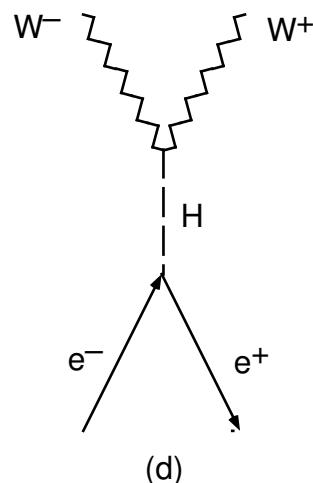
(a)



(b)



(c)

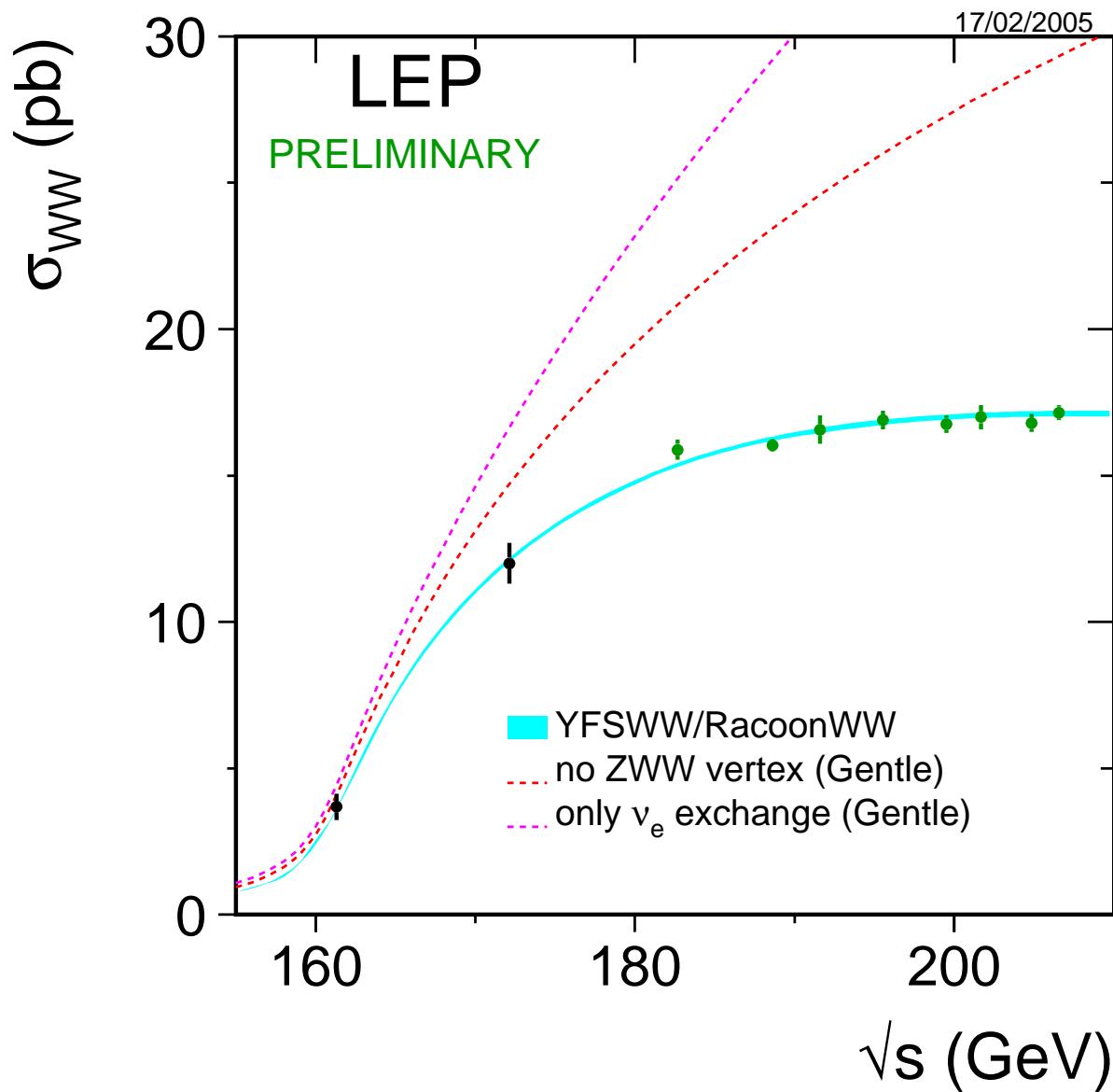


(d)

$J = 1$ partial-wave amplitudes $\mathcal{M}_\gamma^{(1)}$, $\mathcal{M}_Z^{(1)}$, $\mathcal{M}_\nu^{(1)}$
have—individually—unacceptable high-energy
behavior ($\propto s$)

... But sum is well-behaved

“Gauge cancellation” observed at LEP2, Tevatron



$J = 0$ amplitude exists because electrons have mass, and can be found in “wrong” helicity state

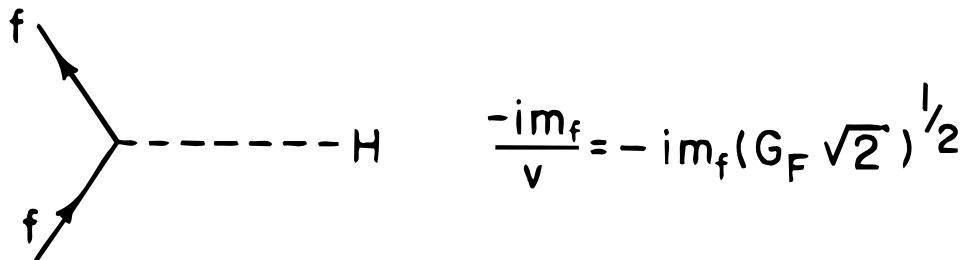
$$\mathcal{M}_\nu^{(0)} \propto s^{\frac{1}{2}} : \text{unacceptable HE behavior}$$

(no contributions from γ and Z)

This divergence is canceled by the Higgs-boson contribution

$\Rightarrow He\bar{e}$ coupling must be $\propto m_e$,

because “wrong-helicity” amplitudes $\propto m_e$



If the Higgs boson did not exist, *something else* would have to cure divergent behavior

IF gauge symmetry were unbroken . . .

- ▷ no Higgs boson
- ▷ no longitudinal gauge bosons
- ▷ no extreme divergences
- ▷ no wrong-helicity amplitudes

 . . . and no viable low-energy phenomenology

In spontaneously broken theory . . .

- ▷ gauge structure of couplings eliminates the most severe divergences
- ▷ lesser—but potentially fatal—divergence arises because the electron has mass
 . . . due to the Higgs mechanism
- ▷ SSB provides its own cure—the Higgs boson

A similar interplay and compensation *must exist* in any acceptable theory

Bounds on M_H

EW theory does not predict Higgs-boson mass

Self-consistency \Rightarrow plausible lower and upper bounds

▷ Conditional *upper bound* from Unitarity

Compute amplitudes \mathcal{M} for gauge boson scattering at high energies, make a partial-wave decomposition

$$\mathcal{M}(s, t) = 16\pi \sum_J (2J + 1) a_J(s) P_J(\cos \theta)$$

Most channels decouple—pw amplitudes are small at all energies (except very near the particle poles, or at exponentially large energies)—for any M_H .

Four interesting channels:

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0 / \sqrt{2} \quad HH / \sqrt{2} \quad HZ_L^0$$

L : longitudinal, $1/\sqrt{2}$ for identical particles

In HE limit,^a s -wave amplitudes $\propto G_F M_H^2$

$$\lim_{s \gg M_H^2} (a_0) \rightarrow \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Require that largest eigenvalue respect the partial-wave unitarity condition $|a_0| \leq 1$

$$\implies M_H \leq \left(\frac{8\pi\sqrt{2}}{3G_F} \right)^{1/2} = 1 \text{ TeV}/c^2$$

condition for perturbative unitarity

^aConvenient to calculate using *Goldstone-boson equivalence theorem*, which reduces dynamics of longitudinally polarized gauge bosons to scalar field theory with interaction Lagrangian given by $\mathcal{L}_{\text{int}} = -\lambda v h(2w^+w^- + z^2 + h^2) - (\lambda/4)(2w^+w^- + z^2 + h^2)^2$, with $1/v^2 = G_F\sqrt{2}$ and $\lambda = G_F M_H^2 / \sqrt{2}$.

- ▷ If the bound is respected
 - ★ weak interactions remain weak at all energies
 - ★ perturbation theory is everywhere reliable

- ▷ If the bound is violated
 - ★ perturbation theory breaks down
 - ★ weak interactions among W^\pm , Z , and H become strong on the 1-TeV scale
 - ⇒ features of *strong* interactions at GeV energies will characterize *electroweak* gauge boson interactions at TeV energies

Threshold behavior of the pw amplitudes a_{IJ} follows from chiral symmetry

$$\begin{aligned}
 a_{00} &\approx G_F s / 8\pi\sqrt{2} && \text{attractive} \\
 a_{11} &\approx G_F s / 48\pi\sqrt{2} && \text{attractive} \\
 a_{20} &\approx -G_F s / 16\pi\sqrt{2} && \text{repulsive}
 \end{aligned}$$

New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV

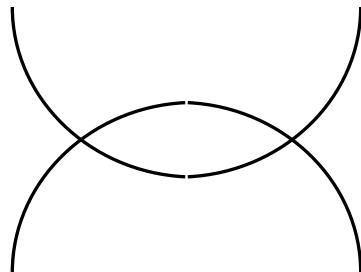
▷ Triviality of scalar field theory

Only *noninteracting* scalar field theories make sense on all energy scales

Quantum field theory vacuum is a dielectric medium that screens charge \Rightarrow *effective charge* is a function of the distance or, equivalently, of the energy scale

running coupling constant

In $\lambda\phi^4$ theory, it is easy to calculate the variation of the coupling constant λ in perturbation theory by summing bubble graphs



$\lambda(\mu)$ is related to a higher scale Λ by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \log(\Lambda/\mu)$$

(Perturbation theory reliable only when λ is small, lattice field theory treats strong-coupling regime)

For stable Higgs potential (*i.e.*, for vacuum energy not to race off to $-\infty$), *require* $\lambda(\Lambda) \geq 0$

Rewrite RGE as an inequality

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log(\Lambda/\mu) .$$

implies an *upper bound*

$$\lambda(\mu) \leq 2\pi^2/3 \log(\Lambda/\mu)$$

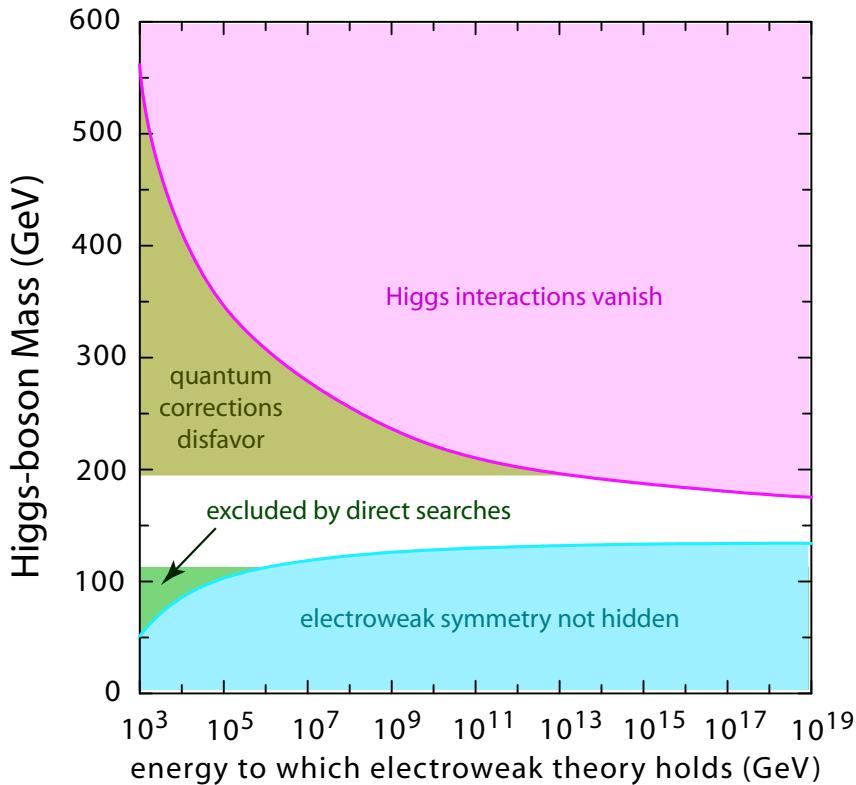
If we require the theory to make sense to arbitrarily high energies—or short distances—then we must take the limit $\Lambda \rightarrow \infty$ while holding μ fixed at some reasonable physical scale. In this limit, the bound forces $\lambda(\mu)$ to zero. \longrightarrow free field theory “trivial”

Rewrite as bound on M_H :

$$\Lambda \leq \mu \exp\left(\frac{2\pi^2}{3\lambda(\mu)}\right)$$

Choose $\mu = M_H$, and recall $M_H^2 = 2\lambda(M_H)v^2$

$$\Lambda \leq M_H \exp\left(4\pi^2 v^2 / 3M_H^2\right)$$



Moral: For any M_H , there is a *maximum energy scale* Λ^* at which the theory ceases to make sense. The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies

Perturbative analysis breaks down when $M_H \rightarrow 1 \text{ TeV}/c^2$ and interactions become strong

Lattice analyses $\implies M_H \lesssim 710 \pm 60 \text{ GeV}/c^2$ if theory describes physics to a few percent up to a few TeV

If $M_H \rightarrow 1 \text{ TeV}$ EW theory lives on brink of instability

- ▷ *Lower bound* by requiring EWSB vacuum
 $V(v) < V(0)$

Requiring that $\langle \phi \rangle_0 \neq 0$ be an absolute minimum of the one-loop potential up to a scale Λ yields the vacuum-stability condition

$$M_H^2 > \frac{3G_F\sqrt{2}}{8\pi^2} (2M_W^4 + M_Z^4 - 4m_t^4) \log(\Lambda^2/v^2)$$

... for $m_t \lesssim M_W$

(No illuminating analytic form for heavy m_t)

If the Higgs boson is relatively light—which would itself require explanation—then the theory can be self-consistent up to very high energies

If EW theory is to make sense all the way up to a unification scale $\Lambda^* = 10^{16}$ GeV, then

$$134 \text{ GeV}/c^2 \lesssim M_H \lesssim 177 \text{ GeV}/c^2$$

Higgs-Boson Properties

$$\Gamma(H \rightarrow f\bar{f}) = \frac{G_F m_f^2 M_H}{4\pi\sqrt{2}} \cdot N_c \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

$\propto M_H$ in the limit of large Higgs mass

$$\Gamma(H \rightarrow W^+W^-) = \frac{G_F M_H^3}{32\pi\sqrt{2}} (1-x)^{1/2} (4-4x+3x^2)$$

$$x \equiv 4M_W^2/M_H^2$$

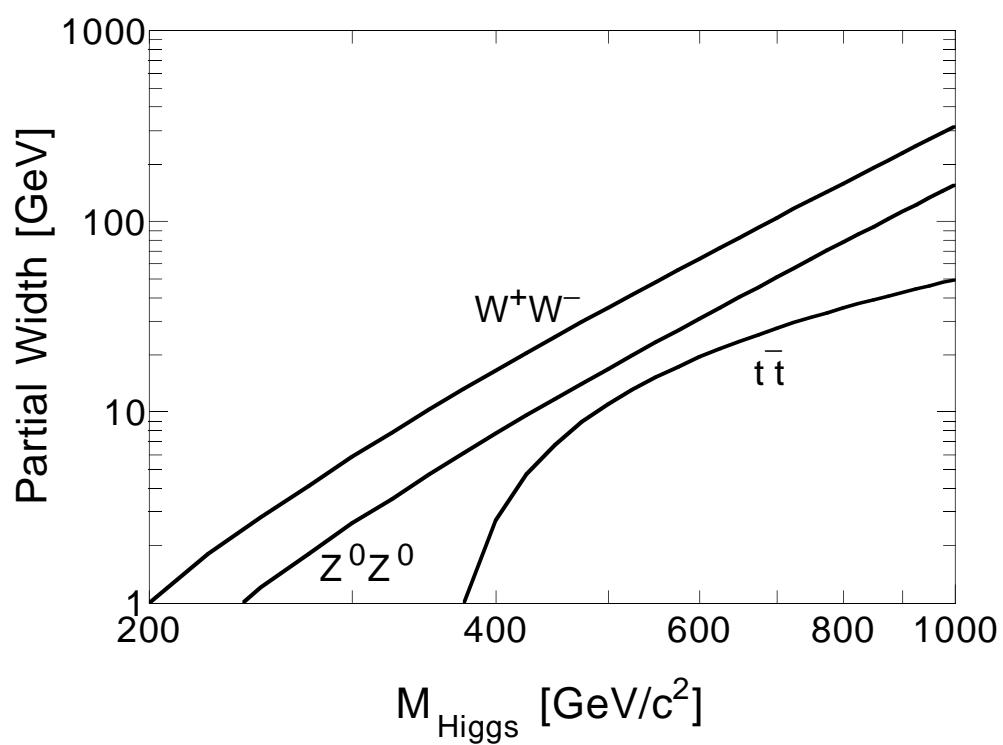
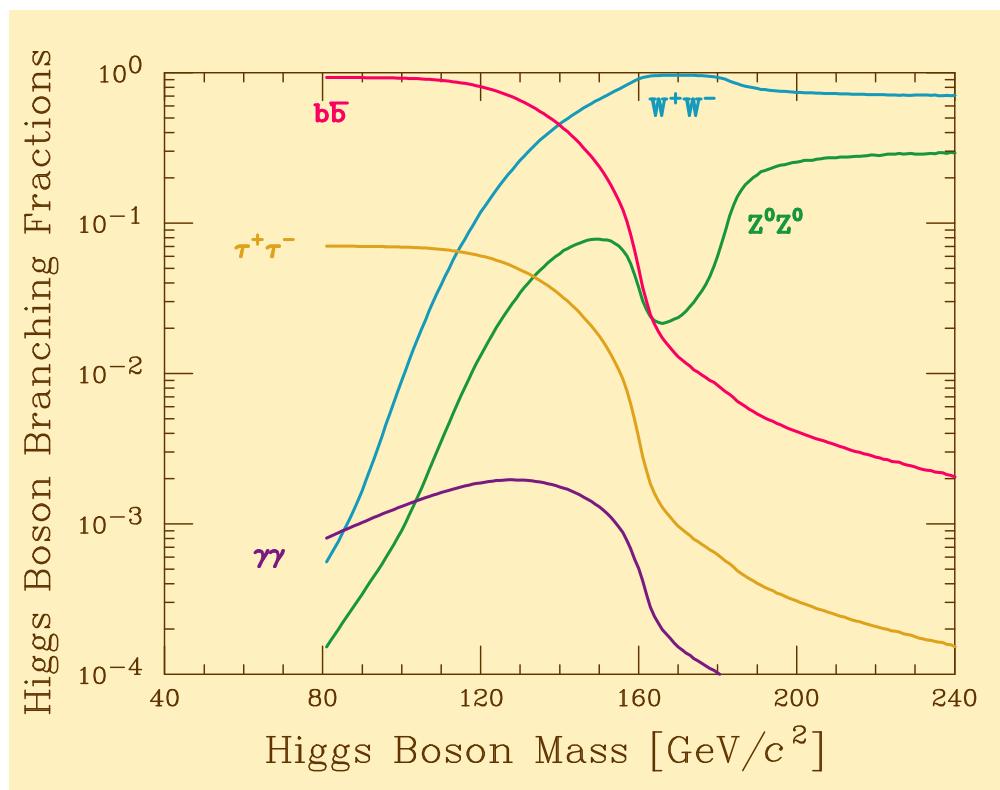
$$\Gamma(H \rightarrow Z^0Z^0) = \frac{G_F M_H^3}{64\pi\sqrt{2}} (1-x')^{1/2} (4-4x'+3x'^2)$$

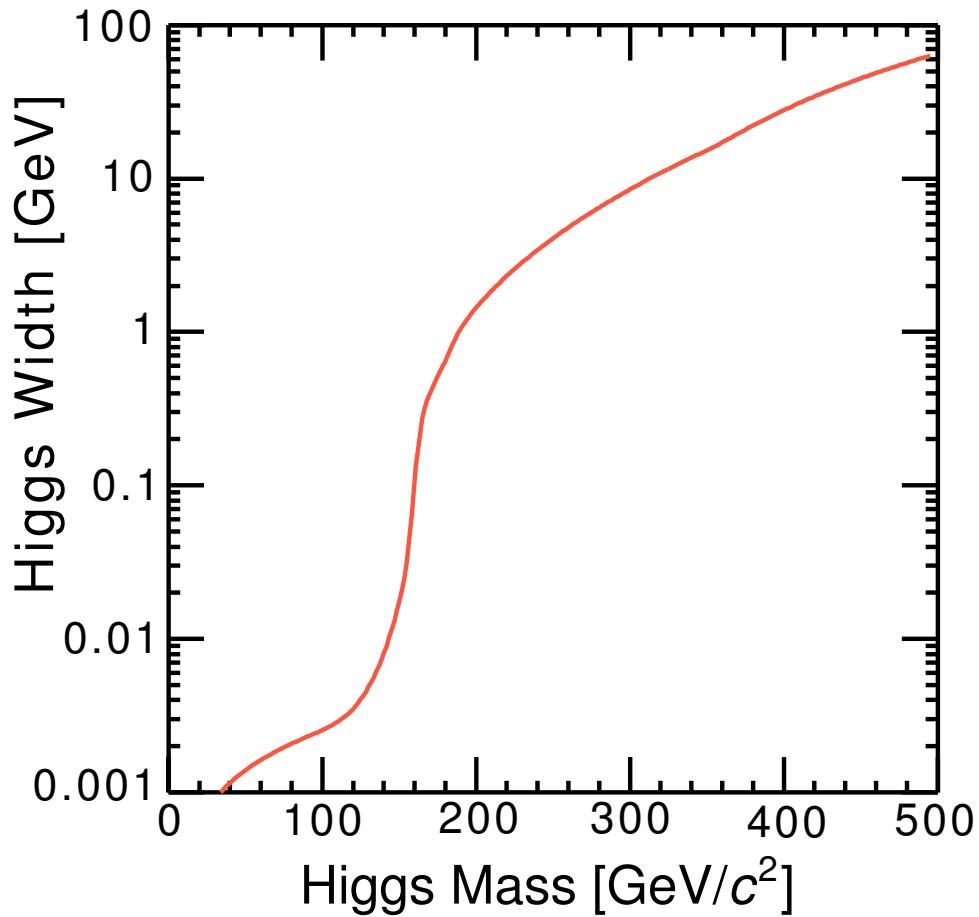
$$x' \equiv 4M_Z^2/M_H^2$$

asymptotically $\propto M_H^3$ and $\frac{1}{2}M_H^3$, respectively
($\frac{1}{2}$ from weak isospin)

$2x^2$ and $2x'^2$ terms \Leftrightarrow decays into transversely polarized gauge bosons

Dominant decays for large M_H into pairs of longitudinally polarized weak bosons





Below W^+W^- threshold, $\Gamma_H \lesssim 1$ GeV

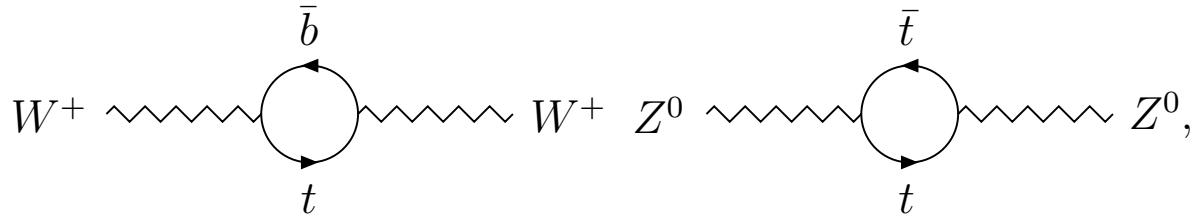
Far above W^+W^- threshold, $\Gamma_H \propto M_H^3$

For $M_H \rightarrow 1$ TeV/c², Higgs boson is an *ephemeron*, with a perturbative width approaching its mass.

Clues to the Higgs-boson mass

Sensitivity of EW observables to m_t gave early indications for massive top

quantum corrections to SM predictions for M_W and M_Z arise from different quark loops



... alter link between the M_W and M_Z :

$$M_W^2 = M_Z^2 (1 - \sin^2 \theta_W) (1 + \Delta\rho)$$

where $\Delta\rho \approx \Delta\rho^{(\text{quarks})} = 3G_F m_t^2 / 8\pi^2 \sqrt{2}$

strong dependence on m_t^2 accounts for precision of m_t estimates derived from EW observables

m_t known to $\pm 1.33\%$ from Tevatron ...

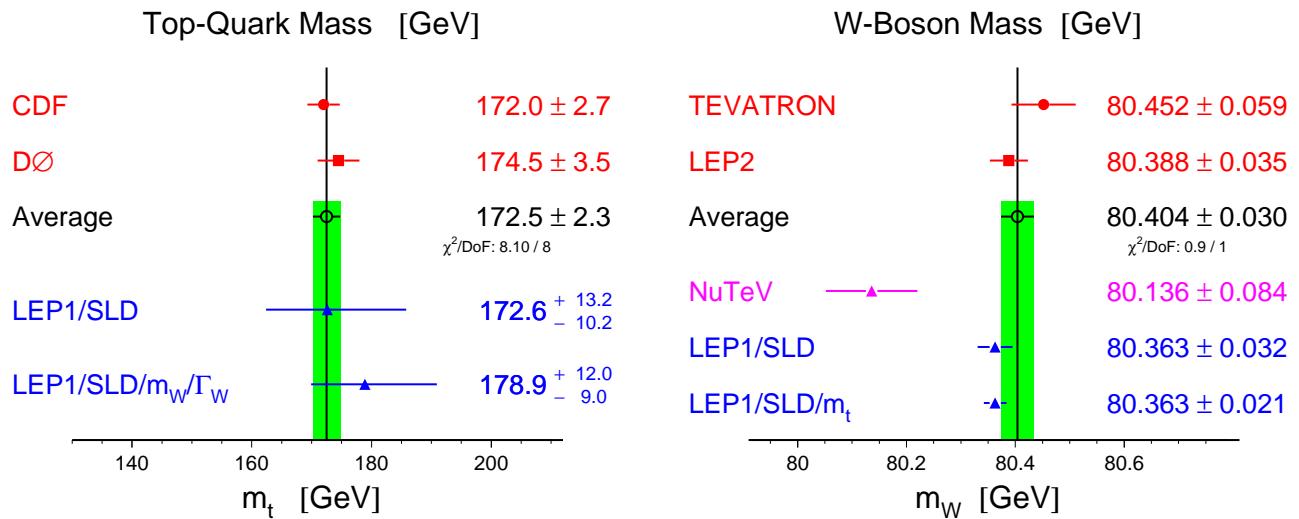
⇒ look beyond the quark loops to next most important quantum corrections:

Higgs-boson effects

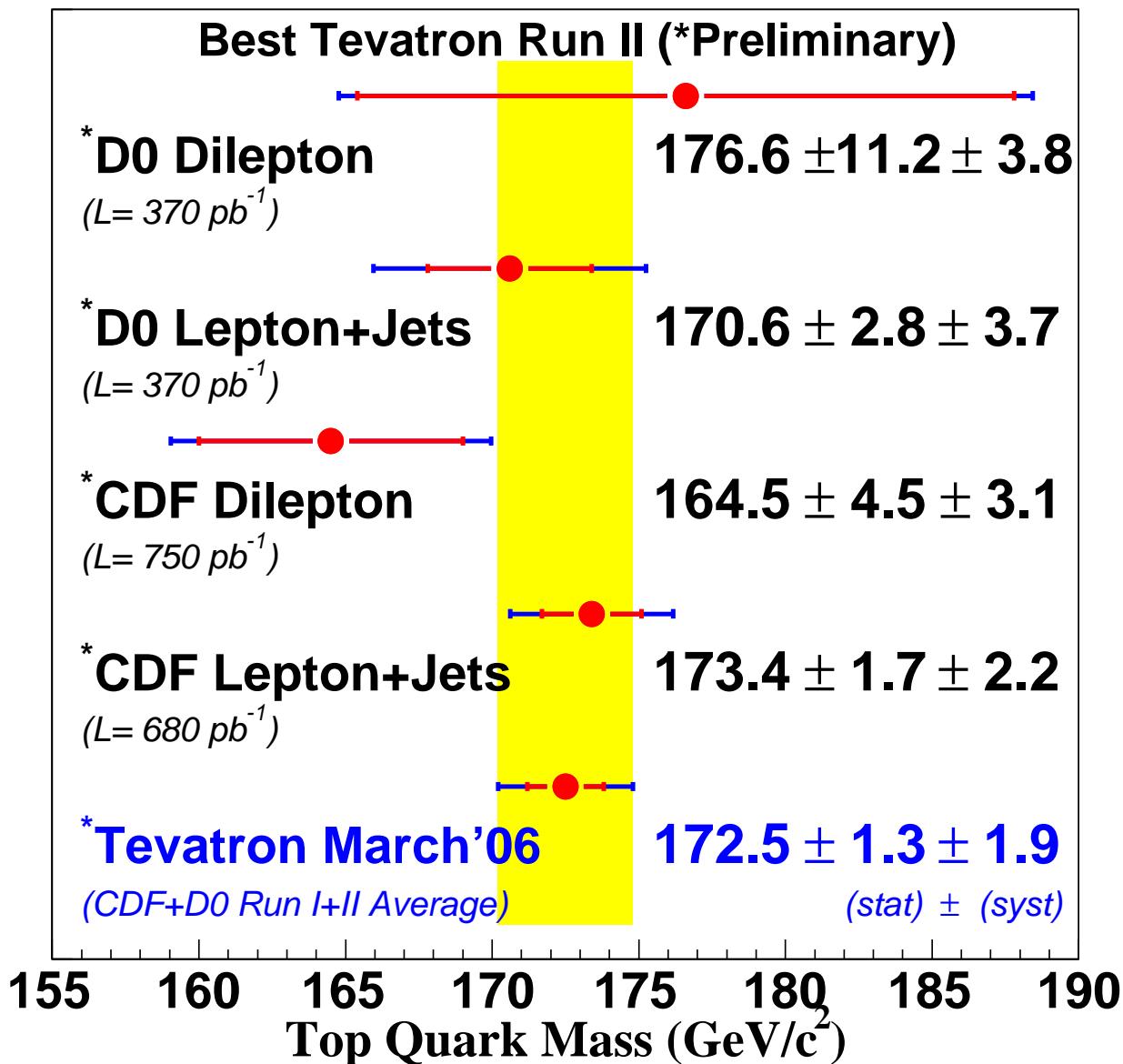
H quantum corrections smaller than t corrections, exhibit more subtle dependence on M_H than the m_t^2 dependence of the top-quark corrections

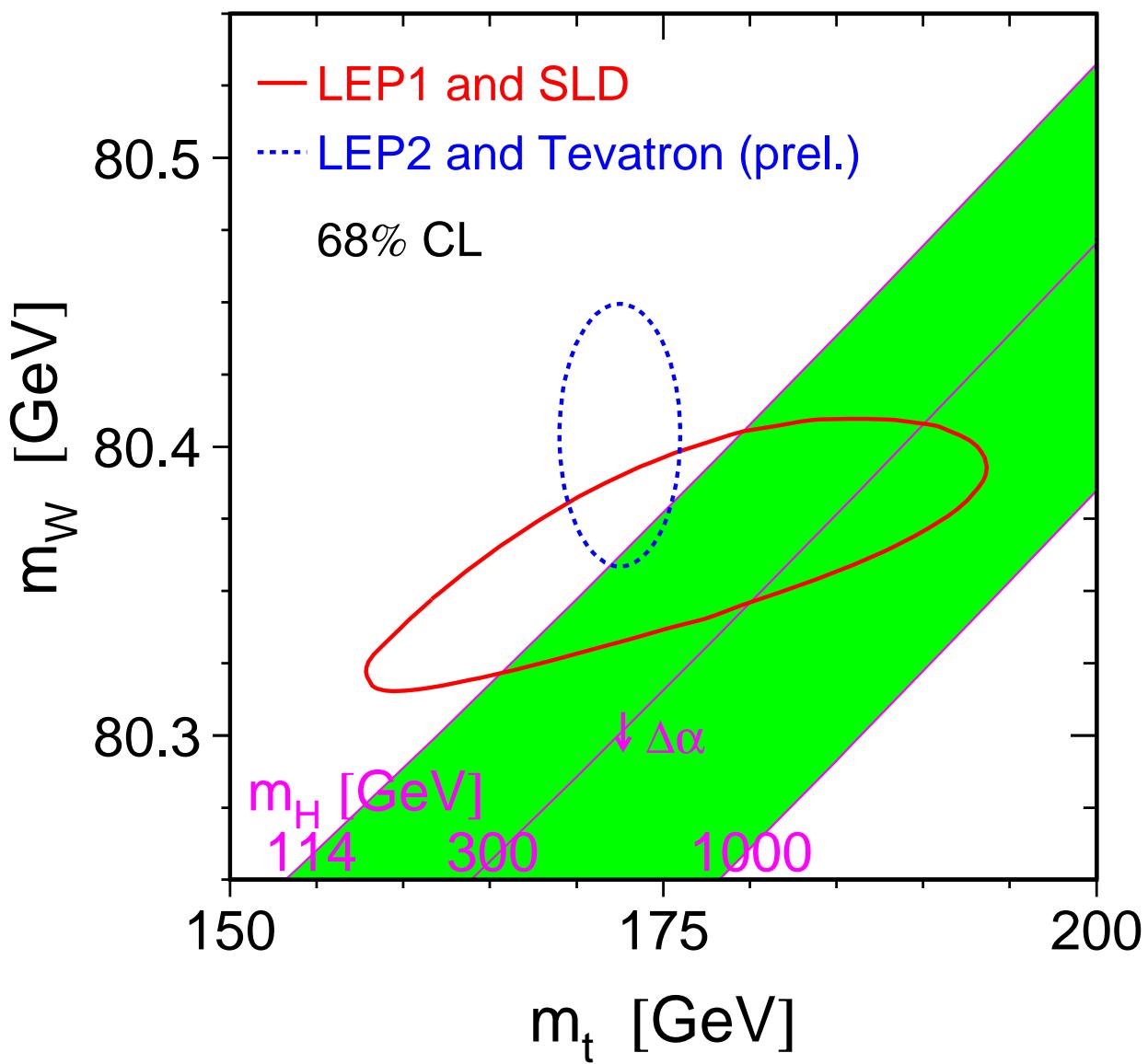
$$\Delta\rho^{(\text{Higgs})} = \mathcal{C} \cdot \ln\left(\frac{M_H}{v}\right)$$

M_Z known to 23 ppm, m_t and M_W well measured



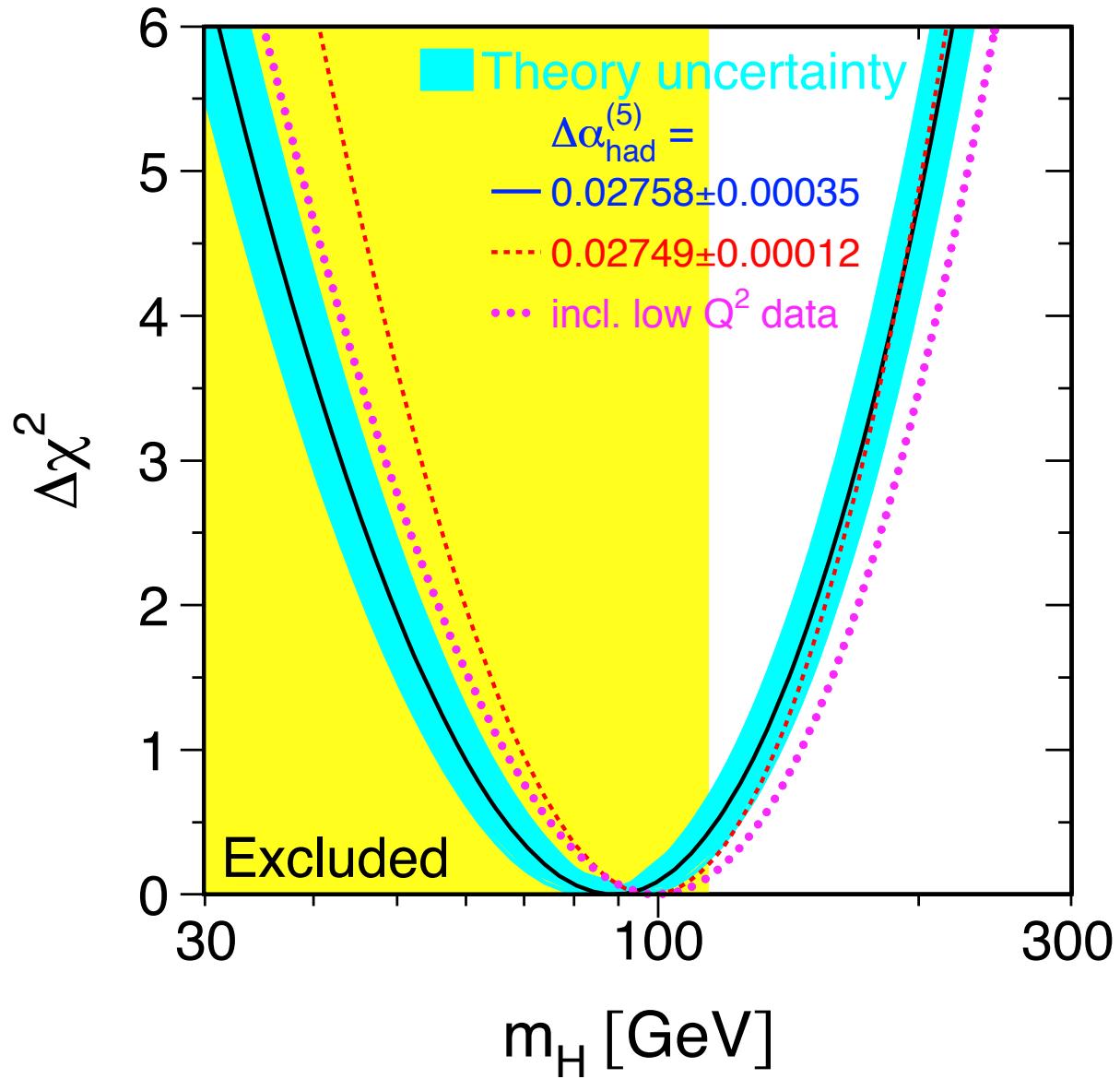
so examine dependence of M_W upon m_t and M_H





Direct, indirect determinations agree reasonably
 Both favor a light Higgs boson,
within framework of SM analysis.

Fit to a universe of data



Standard-Model $M_H \lesssim 207$ GeV at 95% CL

Within SM, LEPEWWG deduce a 95% CL upper limit, $M_H \lesssim 207 \text{ GeV}/c^2$.

Direct searches at LEP $\Rightarrow M_H > 114.4 \text{ GeV}/c^2$, excluding much of the favored region

either the Higgs boson is just around the corner, or SM analysis is misleading

Things will soon be popping!

Expect progress from M_W - m_t - M_H correlation

- ▷ Tevatron and LHC measurements will determine m_t within 1 or 2 GeV/c^2
- ▷ ... and improve δM_W to about 15 MeV/c^2
- ▷ As the Tevatron's integrated luminosity approaches 10 fb^{-1} , CDF and DØ will begin to explore the region of M_H not excluded by LEP
- ▷ ATLAS and CMS will carry on the exploration of the Higgs sector at the LHC

A few words on Higgs production . . .

$e^+e^- \rightarrow H$: hopelessly small

$\mu^+\mu^- \rightarrow H$: scaled by $(m_\mu/m_e)^2 \approx 40\,000$

$e^+e^- \rightarrow HZ$: prime channel

Hadron colliders:

$gg \rightarrow H \rightarrow b\bar{b}$: background ?!

$gg \rightarrow H \rightarrow \gamma\gamma$: rate ?!

$\bar{p}p \rightarrow H(W, Z)$: prime Tevatron channel

At the LHC:

Many channels become accessible, expect sensitive search up to 1 TeV

Aside: varieties of neutrino mass

Chiral decomposition of Dirac spinor:

$$\psi = \frac{1}{2}(1 - \gamma_5)\psi + \frac{1}{2}(1 + \gamma_5)\psi \equiv \psi_L + \psi_R$$

$$\psi^c \equiv C\bar{\psi}^T; \quad C = i\gamma^2\gamma^0$$

Charge conjugate of RH field is LH:

$$\psi_L^c \equiv (\psi^c)_L = (\psi_R)^c$$

Possible forms for mass terms

Dirac connects LH, RH components of *same field*

$$\mathcal{L}_D = D(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) = D\bar{\psi}\psi$$

$$\implies \text{mass eigenstate } \psi = \psi_L + \psi_R$$

(invariant under global phase rotation $\nu \rightarrow e^{i\theta}\nu$,
 $\ell \rightarrow e^{i\theta}\ell$, so that lepton number is conserved)

Possible forms for mass terms (cont'd)

Majorana connects LH, RH components of
conjugate fields

$$\begin{aligned}-\mathcal{L}_{\text{MA}} &= A(\bar{\psi}_R^c \psi_L + \bar{\psi}_L \psi_R^c) = A\bar{\chi}\chi \\ -\mathcal{L}_{\text{MB}} &= B(\bar{\psi}_L^c \psi_R + \bar{\psi}_R \psi_L^c) = B\bar{\omega}\omega\end{aligned}$$

for which the mass eigenstates are

$$\begin{aligned}\chi &\equiv \psi_L + \psi_R^c = \chi^c = \psi_L + (\psi_L)^c \\ \omega &\equiv \psi_R + \psi_L^c = \omega^c = \psi_R + (\psi_R)^c\end{aligned}$$

\mathcal{L}_M violates lepton number by two units
⇒ Majorana ν can mediate $\beta\beta_{0\nu}$ decays



Detecting $\beta\beta_{0\nu}$ would offer decisive evidence for the Majorana nature of ν

EWSB: another path?

Modeled EWSB on Ginzburg–Landau description of SC phase transition

had to introduce new, elementary scalars

GL is not the last word on superconductivity:
dynamical Bardeen–Cooper–Schrieffer theory

The elementary fermions—electrons—and gauge interactions—QED—needed to generate the scalar bound states are already present in the case of superconductivity. Could a scheme of similar economy account for EWSB?

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y + \text{massless } u \text{ and } d$$

Treat $SU(2)_L \otimes U(1)_Y$ as perturbation

$m_u = m_d = 0$: QCD has exact $SU(2)_L \otimes SU(2)_R$ chiral symmetry. At an energy scale $\sim \Lambda_{\text{QCD}}$, strong interactions become strong, fermion condensates appear, and $SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V$
⇒ 3 Goldstone bosons, one for each broken generator: 3 massless pions (Nambu)

Broken generators: 3 axial currents; couplings to π
measured by pion decay constant f_π

Turn on $SU(2)_L \otimes U(1)_Y$: EW gauge bosons couple
to axial currents, acquire masses of order $\sim g f_\pi$

$$\mathcal{M}^2 = \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & gg' \\ 0 & 0 & gg' & g'^2 \end{pmatrix} \frac{f_\pi^2}{4},$$

$$(W^+, W^-, W_3, \mathcal{A})$$

same structure as standard EW theory. Diagonalize:
 $M_W^2 = g^2 f_\pi^2 / 4$, $M_Z^2 = (g^2 + g'^2) f_\pi^2 / 4$, $M_A^2 = 0$, so

$$\frac{M_Z^2}{M_W^2} = \frac{(g^2 + g'^2)}{g^2} = \frac{1}{\cos^2 \theta_W}$$

Massless pions disappear from physical spectrum, to
become longitudinal components of weak bosons

$M_W \approx 30 \text{ MeV}/c^2$

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$$M_W \approx 30 \text{ MeV}/c^2$$

No fermion masses . . .

With no Higgs mechanism . . .

- ▷ Quarks and leptons would remain massless
- ▷ QCD would confine them in color-singlet hadrons
- ▷ *Nucleon mass would be little changed*, but proton outweighs neutron
- ▷ QCD breaks EW symmetry, gives $(1/2500 \times \text{observed})$ masses to W , Z , so weak-isospin force doesn't confine
- ▷ Rapid! β -decay \Rightarrow lightest nucleus is one neutron; no hydrogen atom
- ▷ Probably some light elements in BBN, but ∞ Bohr radius
- ▷ No atoms (as we know them) means no chemistry, no stable composite structures like the solids and liquids we know

. . . the character of the physical world would be profoundly changed

Assessment

$SU(2)_L \otimes U(1)_Y$: 25 years of confirmations

- ★ neutral currents; W^\pm , Z^0
- ★ charm

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★ neutral currents; W^\pm , Z^0

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(+ experimental guidance)

★ τ , ν_τ

★ b , t

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$SU(2)_L \otimes U(1)_Y$: 25 years of confirmations

★ neutral currents; W^\pm , Z^0

★ charm

(+ experimental guidance)

★ τ , ν_τ

★ b , t

+ experimental surprises

★ narrowness of ψ , ψ'

★ long B lifetime; large $B^0 - \bar{B}^0$ mixing

★ heavy top

10 years of precision measurements . . .

. . . find no significant deviations

quantum corrections tested at $\pm 10^{-3}$

No “new physics” . . . yet!

Theory tested at distances

from 10^{-17} cm

to $\sim 10^{22}$ cm

origin Coulomb’s law (tabletop experiments)

smaller $\left\{ \begin{array}{l} \text{Atomic physics} \rightarrow \text{QED} \\ \text{high-energy expts.} \rightarrow \text{EW theory} \end{array} \right.$

larger $M_\gamma \approx 0$ in planetary . . . measurements

Is EW theory true?

Complete ??

The EW scale and beyond

EWSB scale, $v = (G_F \sqrt{2})^{-\frac{1}{2}} \approx 246$ GeV,
sets

$$M_W^2 = g^2 v^2 / 2 \quad M_Z^2 = M_W^2 / \cos^2 \theta_W$$

But it is not the only scale of physical interest

quasi-certain: $M_{\text{Planck}} = 1.22 \times 10^{19}$ GeV

probable: $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$
unification scale $\sim 10^{15-16}$ GeV

somewhere: flavor scale

How to keep the distant scales from mixing in the face of quantum corrections?

OR

How to stabilize the mass of the Higgs boson on the electroweak scale?

OR

Why is the electroweak scale small?

“The hierarchy problem”

Higgs potential $V(\phi^\dagger \phi) = \mu^2 (\phi^\dagger \phi) + |\lambda| (\phi^\dagger \phi)^2$

$\mu^2 < 0$: $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{em}}$, as

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ \sqrt{-\mu^2/2|\lambda|} \end{pmatrix} \equiv \begin{pmatrix} 0 \\ \underbrace{(G_F \sqrt{8})^{-1/2}}_{175 \text{ GeV}} \end{pmatrix}$$

Beyond classical approximation, quantum corrections to scalar mass parameters:

$$m^2(p^2) = m_0^2 + \text{---} \xrightarrow{\text{J=1}} + \text{---} \xrightarrow{\text{J=1/2}} + \text{---} \xrightarrow{\text{J=0}}$$

Loop integrals are potentially divergent.

$$m^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \dots$$

Λ : reference scale at which m^2 is known

g : coupling constant of the theory

C : coefficient calculable in specific theory

For the mass shifts induced by radiative corrections to remain under control (not greatly exceed the value measured on the laboratory scale), *either*

- ▷ Λ must be small, or
- ▷ new physics must intervene to cut off integral

BUT natural reference scale for Λ is

$$\Lambda \sim M_{\text{Planck}} = \left(\frac{\hbar c}{G_{\text{Newton}}} \right)^{1/2} \approx 1.22 \times 10^{19} \text{ GeV}$$

for $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

OR

$$\Lambda \sim M_U \approx 10^{15}\text{-}10^{16} \text{ GeV}$$

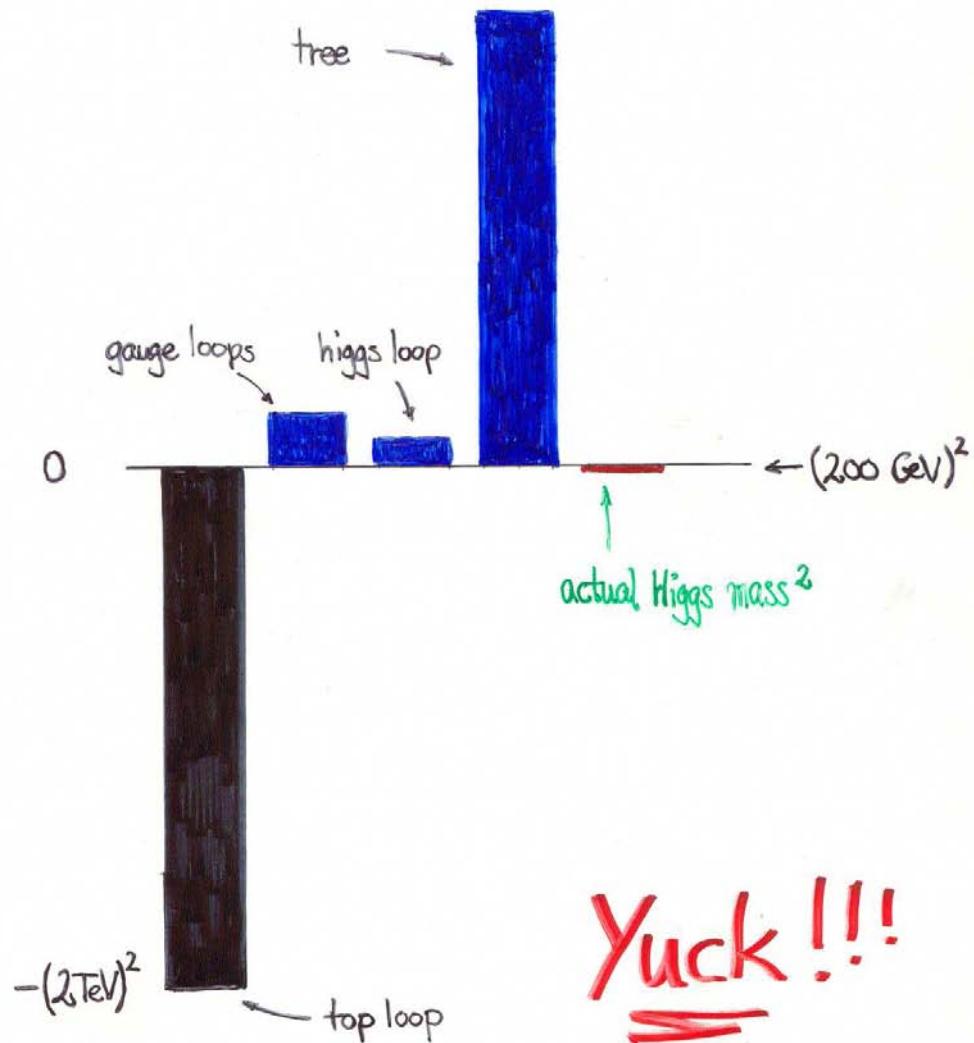
for unified theory

Both $\gg v/\sqrt{2} \approx 175 \text{ GeV} \implies$

New Physics at $E \lesssim 1 \text{ TeV}$

Fine tuning the Higgs

$\Lambda = 10 \text{ TeV}$



Martin Schmaltz, ICHEP02

Only a few distinct scenarios . . .

- ▷ Supersymmetry: balance contributions of fermion loops (-1) and boson loops $(+1)$

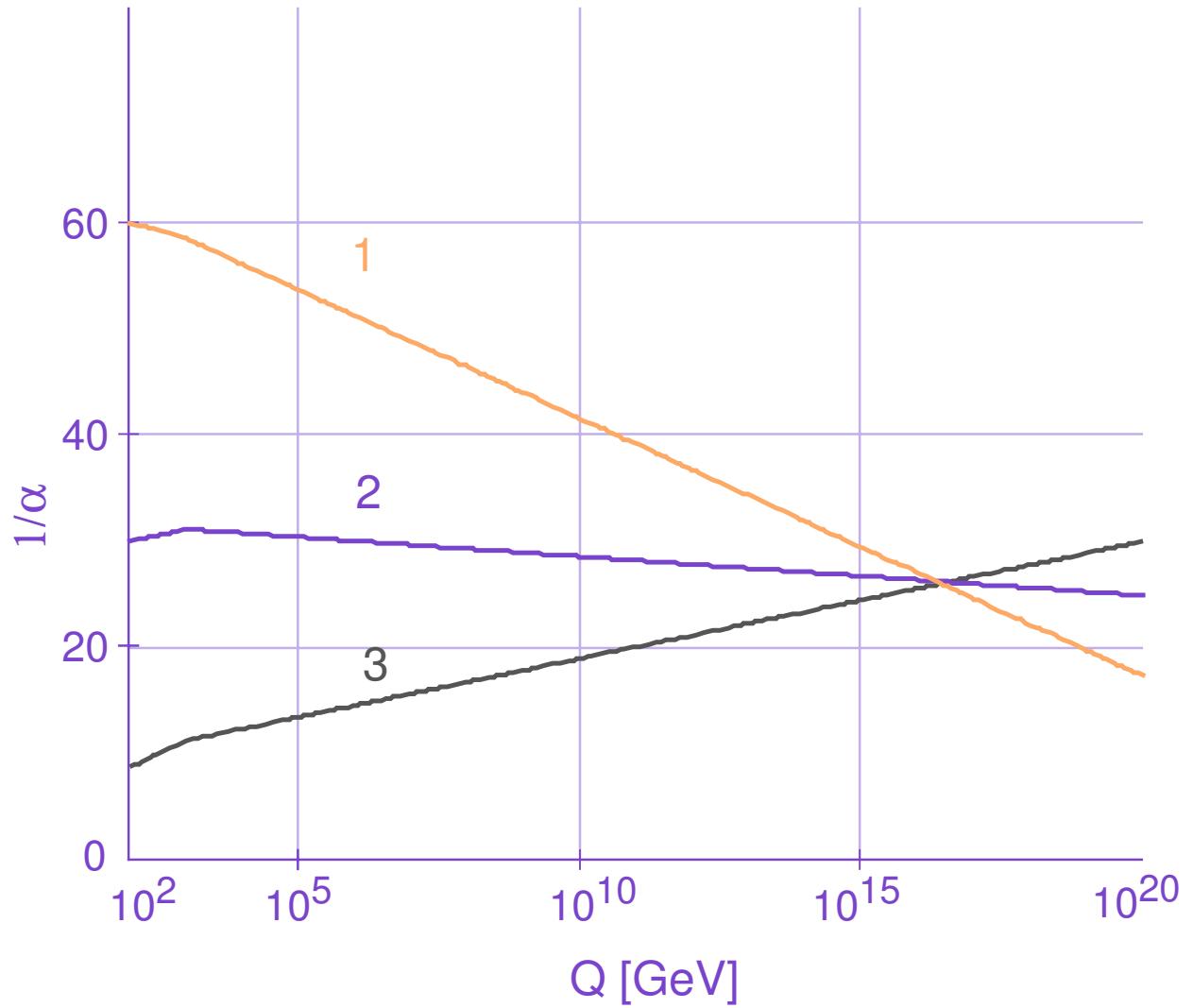
Exact supersymmetry,

$$\sum_{\substack{i= \\ \text{fermions} \\ + \text{bosons}}} C_i \int dk^2 = 0$$

Broken supersymmetry, shifts acceptably small if superpartner mass splittings are not too large

$g^2 \Delta M^2$ “small enough” $\Rightarrow \widetilde{M} \lesssim 1 \text{ TeV}/c^2$

Coupling constant unification?



Only a few distinct scenarios . . .

- ▷ Composite scalars (technicolor): New physics arises on scale of composite Higgs-boson binding,

$$\Lambda_{\text{TC}} \simeq O(1 \text{ TeV})$$

“Form factor” cuts effective range of integration

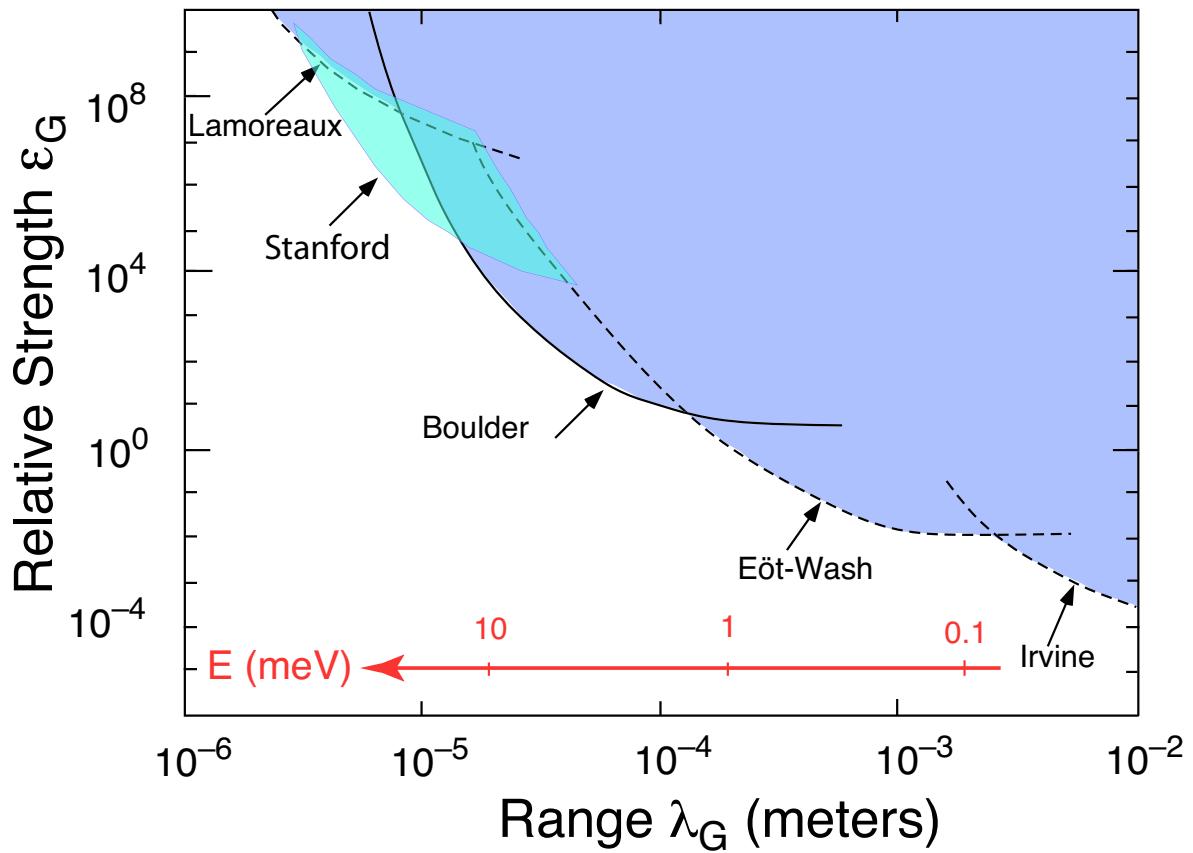
- ▷ Strongly interacting gauge sector: WW resonances, multiple W production, probably scalar bound state “quasiHiggs” with $M < 1 \text{ TeV}$

Only a few distinct scenarios . . .

- ▷ Extra spacetime dimensions:
pseudo-Nambu–Goldstone bosons, extra
particles to cancel integrand, . . .
- ▷ Planck mass is a mirage, based on a false
extrapolation of Newton’s $1/r^2$ force law

Gravity follows $1/r^2$ law down to $\lesssim 1$ mm
 (few meV)

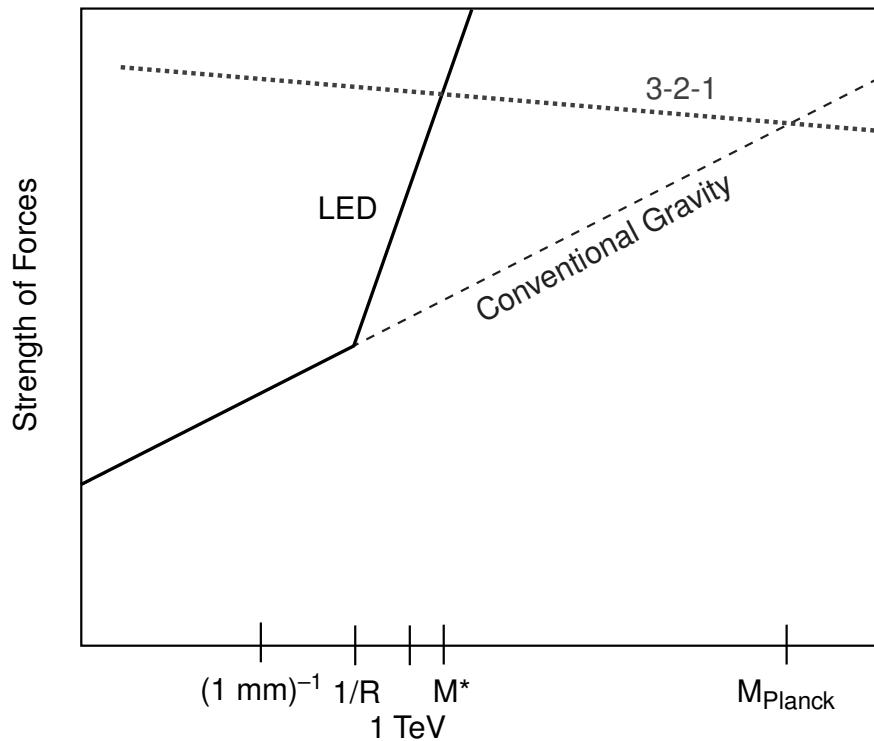
$$V(r) = - \int dr_1 \int dr_2 \frac{G_N \rho(r_1) \rho(r_2)}{r_{12}} [1 + \varepsilon_G \exp(-r_{12}/\lambda_G)]$$



Experiment leaves us free to consider modifications to Gravity even at (nearly) macroscopic distances

Suppose at scale R Gravity propagates in $3 + n$ spatial dimensions

Force law changes: $F \propto 1/r^{2+n}$



$$G_N \sim M_{\text{Pl}}^{-2} \sim M^{*-n-2} R^{-n}$$

M^* : gravity's true scale

Example:

$$M^* = 1 \text{ TeV}$$

$$\Rightarrow R \lesssim 10^{-3} \text{ m for } n = 2$$

M_{Pl} is a mirage (false extrapolation)!

Why the LHC is so exciting (I)

- ▷ Even low luminosity opens vast new realm: 10 pb^{-1} (*few days at initial \mathcal{L}*) yields
8000 top quarks, $10^5 W$ -bosons,
100 QCD dijets beyond Tevatron kinematic limit
- ▷ The antithesis of a one-experiment machine; enormous scope and versatility beyond high- p_\perp

Why the LHC is so exciting (II)

- ▷ Electroweak theory (unitarity argument) tells us the 1-TeV scale is special: Higgs boson or other new physics (strongly interacting gauge bosons)
- ▷ Hierarchy problem \Rightarrow other new physics nearby
- ▷ Our ignorance of EWSB obscures our view of other questions (identity problem, for example).
Lifting the veil at 1 TeV will change the face of physics

High expectations for the Tevatron

- ▷ Biggest changes in the way we think about LHC experiments have come from the Tevatron: the large mass of the top quark and the success of silicon microvertex detectors: **heavy flavors**
- ▷ Top quark is a unique window on EWSB and of interest in its own right: **single top production**
- ▷ Entering new terrain for new gauge bosons, strong dynamics, SUSY, Higgs, B_s mixing, . . .